1. Consider an A-B binary single-phase alloy with n_A A atoms and n_B B atoms. According to statistical mechanics, the configurational entropy change (ΔS) relative to pure A and pure B can be expressed as

$$\Delta S = -k\left[n_A \ln \frac{n_A}{n} + n_B \ln \frac{n_B}{n} \right],$$

where k is Boltzmann constant, and $n = n_A + n_B$. Derive the above equation, including the approximation(s) if any, step by step. (10%)

2. Anneal a long steel rod of radius r_s in a furnace. The temperature (T) on the surface of the rod ($r = r_s$) is always maintained constant ($T = T_s$). In the center of the rod the temperature rises fom T_0, with $\frac{\partial T}{\partial r}$ being equal to zero all the time. The temperature variation of the rod as a function of time (t) is described by

$$\frac{\partial T}{\partial t} = \frac{k}{c\rho}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \right),$$

where k, c and ρ are thermal conductivity, specific heat and density of the rod, respectively. Solve the above differential equation. (Hint: find Bessel differential equation). (15%)

3. Evaluate the integral $\int_0^\infty (e^x - 1)^{-1} x^3 \, dx$. (10%)

4. Stress-strain relation, in terms of linear elasticity, can be expressed as the matrix form given in the next page if the strained body is isotropic. Prove that $2C_{44} = C_{11} - C_{12}$. In other words, there exist only two independent elastic constants for isotropic materials.
Hint: try to rotate the coordinate system.

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{pmatrix} \begin{pmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{23} \\ 2\epsilon_{31} \\ 2\epsilon_{12} \end{pmatrix},$$

where σ_{ij}, C_{ij} and ϵ_{ij} are stresses, elastic constants and strains, respectively. The strains are disignated with tensor notations. Namely, $\epsilon_{ij}\,(i \neq j) = \frac{1}{2}\gamma_{ij}\,(i \neq j)$, where γ_{ij} are conventional engineering strains. (15%)

1. An elevator and its load have a total mass of 900 kg. Find the tension T in the supporting cable when the elevator, originally moving downward at 15 $m.s^{-1}$, is brought to rest with constant acceleration in a distance of 30 m.

2. A steel bar 15 cm long is welded end-to-end to a copper bar 30 cm long. Each bar has a square cross section, 3 cm on a side. The free end of the steel bar is placed in contact with steam at 100°C, and the free end of the copper bar with ice at 0°C. Find the temperature at the junction of the two bars and the total rate of heat flow, when steady-state conditions have been reached. k_s=50.2 $J.S^{-1}m^{-1}.(°C)^{-1}$, k_c=385 $J.S^{-1}, m^{-1}(°C)^{-1}$

3. A particle having a charge q=8 x 10^{-9}C moves from point a to point b along a straight line, a total distance d=0.8m. The electric field is uniform along this line, in the direction from a to b, with magnitude E=300$N.C^{-1}$. Determine the force on q, the work done on it by the field, and the potential difference Va-Vb.

4. If the current in the coil with self-inductance 20μH increases uniformly from zero to 2 A in 0.5 S, find the magnitude and direction of the self-induced emf.

5. A certain 60-watt lightbulb emits a total luminous flux of 1000 lm, distributed uniformly over a hemisphere. Find the illuminance and the luminous intensity at a distance of 2 m.