

1. Evaluate the integral  $\int_{-\infty}^{\infty} e^{-x^2} dx$ . (10%)
2. Evaluate the integral  $\int_0^{\infty} e^{-x} x^n dx$ . (15%)
3. Prove that  $\ln(n!) \cong n \ln n - n$ , where  $n$  is an integer and  $n \gg 1$ . (10%)
4. Given that  $dy/dx = x + y$ , solve  $y$  as a function of  $x$  in which  $y(x=0) = 1$ . (15%)

1. An elevator and its load have a total mass of 900 kg. Find the tension  $T$  in the supporting cable when the elevator, originally moving downward at  $15 \text{ m}\cdot\text{s}^{-1}$ , is brought to rest with constant acceleration in a distance of 30 m.
2. A steel bar 15 cm long is welded end-to-end to a copper bar 30 cm long. Each bar has a square cross section, 3 cm on a side. The free end of the steel bar is placed in contact with steam at  $100^\circ\text{C}$ , and the free end of the copper bar with ice at  $0^\circ\text{C}$ . Find the temperature at the junction of the two bars and the total rate of heat flow, when steady-state conditions have been reached.  $k_s = 50.2 \text{ J}\cdot\text{s}^{-1}\cdot\text{m}^{-1}\cdot(^{\circ}\text{C})^{-1}$ ,  $k_c = 385 \text{ J}\cdot\text{s}^{-1}\cdot\text{m}^{-1}\cdot(^{\circ}\text{C})^{-1}$
3. A particle having a charge  $q = 8 \times 10^{-9} \text{ C}$  moves from point a to point b along a straight line, a total distance  $d = 0.8 \text{ m}$ . The electric field is uniform along this line, in the direction from a to b, with magnitude  $E = 300 \text{ N}\cdot\text{C}^{-1}$ . Determine the force on  $q$ , the work done on it by the field, and the potential difference  $V_a - V_b$ .
4. If the current in the coil with self-inductance  $20 \mu\text{H}$  increases uniformly from zero to 2 A in 0.5 S, find the magnitude and direction of the self-induced emf.
5. A certain 60-watt lightbulb emits a total luminous flux of 1000 lm, distributed uniformly over a hemisphere. Find the illuminance and the luminous intensity at a distance of 2 m.