著作物理

1. Vector D is a linear combination of three noncoplanar (and nonorthogonal) vector :

$$D = aA + bB + cC$$

Show that the coefficients are given by a ratio of triple scalar products

$$a = \frac{D\cdot B\times C}{A\cdot B\times C} \quad \text{and so on.}$$ (8%)

2. Find the eigenvalues and an orthonormal (orthogonal and normalized) set of eigenvectors for the matrices:

$$M_x = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad M_y = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix},$$

and

$$M_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$ (15%)

3. In stationary directional solidification, the general differential equation describing linear heat flow in the static system is given by

$$\alpha \frac{d^2T}{dx^2} = \frac{dT}{dt}$$

where $\alpha = K/\rho c$, ρ = density, c = specific heat and K = thermal conductivity. Solve the equation, using the boundary conditions as:

1. $T = T_o$ at $x = 0$ for all t.
2. $T = Tm$ at $x = X$, x is the length of solidified ingot.
3. $K_s\left(\frac{dT}{dx}\right)_{x=X} = \rho L_f V_I$, L_f is the lantent heat of fusion, V_I is the rate of solidification.

(20%)

4. Show that

$$\mathcal{L}^{-1}\left\{\frac{f(s)}{s^2}\right\} = \int_0^t \int_0^v F(u)\,du\,dv.$$ (7%)

5. Consider a solid cylinder of mass M and radius R rolling down an inclined plane without slipping. See fig. 1. Find the speed of its center of mass when the cylinder reaches the bottom?

10%

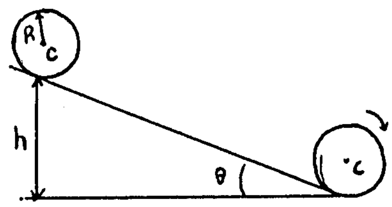

fig. 1.

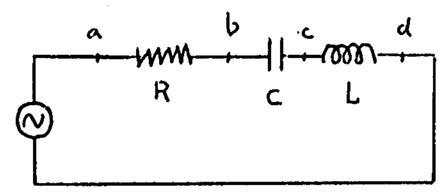

fig. 2.

6. In fig. 2. $R = 15\,\Omega$, $C = 4.7\,\mu F$, and $L = 25\,mH$. The generator provides a sinusoidal voltage of amplitude $\mathcal{E} = 75\,V$ (r.m.s) and frenquency $\nu = 550\,Hz$. (a.) Calculate the r.m.s current amplitude? (b.) Find the r.m.s voltages V_{ab}, V_{bc}, V_{cd}, V_{bd}, V_{ad}? (c.) What average power is dissipated by each of the three circuit elements?

12%

7. An ideal diatomic gas ($\gamma = 1.40$) is caused to pass through the cycle shown on the p-v diagram in fig. 3., where $V_2 = 3V_1$. Determine, in terms of P_1, V_1, T_1, and R: (a.) P_2, P_3, and T_3? (b.) W, Q, ΔU, and ΔS, per mole, for all three processes?

16%

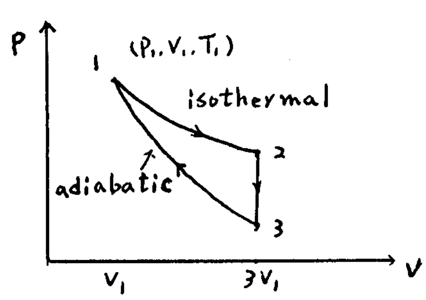

fig. 3.

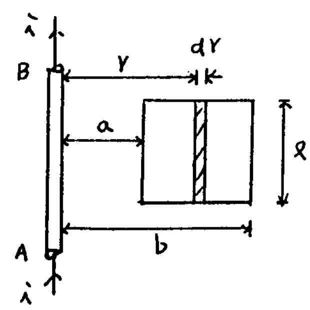

fig. 4.

8. The current in the wire AB of fig. 4. is upward and increasing steadily at a rate di/dt.

12%

(a.) At an instant when the current is i, what are the magnitude and direction of the field $\vec{B}$ at a distance r from the wire?

(b.) What is the flux $d\Phi$ through the narrow shaped strip?

(c.) What is the total flux through the loop?

(d.) What is the induced emf in the loop?