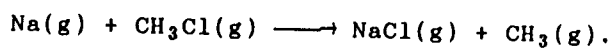


1. The reaction of Na with  $\text{CH}_3\text{Cl}$  by irradiation is according to the equation



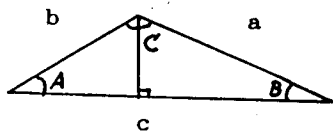
The pressures of Na and  $\text{CH}_3\text{Cl}$  will be adjusted to maintain the size of "flame" (reaction zone) at some convenient magnitude (in radius  $r$ ). The equation of the mass continuity for sodium is as

$$\frac{d^2 p_{\text{Na}}}{dr^2} + \frac{2}{r} \frac{dp_{\text{Na}}}{dr} - \frac{k p_{\text{CH}_3\text{Cl}}}{D_{\text{Na}}} = 0.$$

The minimum value of  $r$  is the nozzle radius  $r_0$ . At this value of radius  $p_{\text{Na}} = p_0$ , and at  $r = R$ ,  $p_{\text{Na}} = p_1$ . What is the  $k$  value? (10%)

2. Express the Laplace's equation  $\partial^2 T / \partial x^2 + \partial^2 T / \partial y^2 = 0$  in polar coordinates and solve it. (15%)

3. Consider the triangle below.



- (a) Show, using elementary trigonometry, that

$$b \cos A + a \cos B = c$$

$$c \cos A + a \cos C = b$$

$$c \cos B + b \cos C = a$$

- (b) If the system of part (a) is thought of as a system of three equations in the three unknowns  $\cos A$ ,  $\cos B$ , and  $\cos C$ , show that the determinant of the system is nonzero.

- (c) Use Cramer's rule to solve for  $\cos C$ .

- (d) Use part (c) to prove the law of cosines:

$$c^2 = a^2 + b^2 - 2ab \cos C. \quad (15\%)$$

4. Find the mass of a metallic sheet in the shape of the hemisphere  $x^2 + y^2 + z^2 = 9$ ,  $z \geq 0$ ,  $x^2 + y^2 \leq 9$ , if its density is proportional to its distance from the origin. (10%)

5. A block of mass  $m = 5 \text{ kg}$  slides down a surface inclined  $37^\circ$  to the horizontal,   
 14% See fig. 1. The coefficient of sliding friction is  $0.25$ . A string attached to the block is wrapped around a flywheel on a fixed axis at  $O$ . The flywheel has a mass  $M = 20 \text{ kg}$ , an outer radius  $R = 0.2 \text{ m}$ , and a moment of inertia with respect to the axis of  $0.2 \text{ kg} \cdot \text{m}^2$ .
- (a.) What is the acceleration of the block down the plane?   
 (b.) What is the tension in the string?

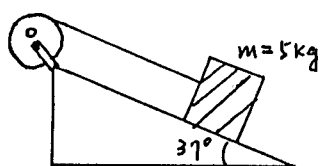


fig. 1.

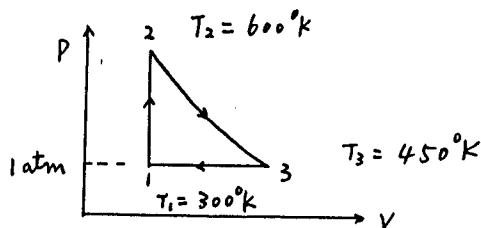


fig. 2.

6. A heat engine takes  $0.1$  mole of an ideal gas around the cycle shown in fig. 2. The value of  $\gamma$  for this gas is  $5/3$ .   
 12% (a.) Find the pressure and volume at points 1, 2, and 3?   
 (b.) Find the net work done by the gas in the cycle?

7. See fig. 3. Find  $\vec{E}(r, \theta)$  and  $V(r, \theta)$  of the electric dipole?   
 10%

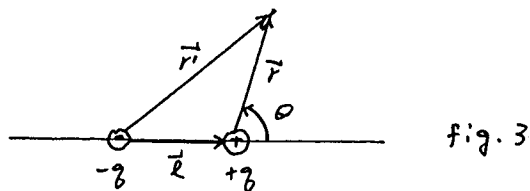


fig. 3

8. A long metal cylinder of radius  $r_a$  is supported on an insulating stand on the axis of a long hollow metal cylinder of inner radius  $r_b$ . The positive charge per unit length on the inner cylinder is  $\lambda$ , and there is an equal negative charge per unit length on the outer cylinder.   
 14% (a.) Calculate the  $\vec{E}(r)$  and  $V(r)$  for   
 ①  $r < r_a$    ②  $r_a < r < r_b$    ③  $r > r_b$
- (b.) ① Calculate the capacitance of a length  $l$  of this capacitor's structure?   
 . ② If  $r_b - r_a = d$ , when  $d \ll r_a$ , discuss the result about ①?