1. Solve the following equations: 50%

(A) $y' = ty/2$, with $y(0) = 1$;

(B) $y'' + y = \tan t$;

(C) $y'' + 2y - 2y^3 = 0$, with $y(0) = 0$, $y'(0) = 1$;

(D) $\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{-(x^2 + y^2)}$;

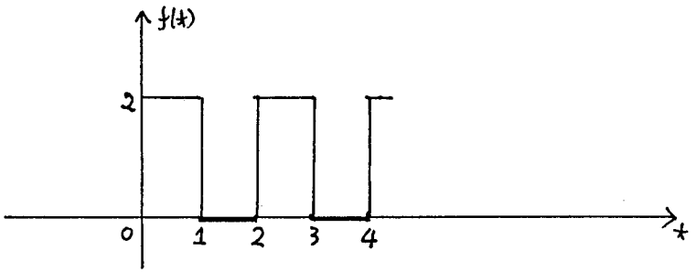
(E) $y' + 2y + 2\int_0^t y(s)\,ds = u(t-2)$, with $y(0) = 1$, where u is the unit step function.

2. Use the method of variation of parameters to solve 20%

$$y'' + 2ay' + a^2y = f(t),$$

where a is a constant and y is a function of t. Use the method of convolution integral to verify that your answer is correct.

3. Find the complex form of the Fourier series of the following shown periodic function f: 10%



4. Suppose that a and b are constants such that the roots of 10%

$$t^2 + at + b = 0,$$

have negative real parts. Prove that every solution $y(x)$ of

$$y'' + ay' + by = 0,$$

satisfies $\lim_{x \to +\infty} y(x) = 0$.

5. Use the method of Frobenius (or the series method) to solve 10%

$$4xy'' + 2y' + y = 0.$$