

1. Solve $y' + 2xy = x$ with $y(0) = 0$. (15%)
2. Use the method of variation of parameters to determine the general solution of $y'' + y = \sec x$, $(0 < x < \frac{\pi}{2})$. (20%)
3. Determine the general solution of $x^2 y'' - 2y = \ln x$, $(x > 0)$. (15%)
4. Suppose x and y are functions of t . Solve the system of differential equations:

$$\begin{cases} (D+3)x + Dy = \cos t \\ (D-1)x + y = \sin t \\ x(0) = 0 \\ y(0) = 4 \end{cases} \quad (15\%)$$

5. Use Laplace transforms to solve

$$y'(t) + 5 \int_0^t \cos 2(t-u)y(u) du = 10 \quad \text{with } y(0) = 2. \quad (15\%)$$

Table of Laplace Transforms

$F(x)$	$f(s) = \mathcal{L}[F(x)]$
1	$\frac{1}{s}$
x^n	$\frac{n!}{s^{n+1}}$
$\sin(bt)$	$\frac{b}{s^2 + b^2}$
$\cos(bt)$	$\frac{s}{s^2 + b^2}$

6. Let R be the radius of the Earth and g the gravitational acceleration at the surface of the Earth. A body of constant mass m is projected upward from the earth's surface with an initial velocity v_0 . Assume there is no air resistance, but taking into consideration the variation of the earth's gravitational field with altitude, find the smallest initial velocity for which the body will not return to the Earth. (20%)