國立成功大學八十學年度礦冶及材料科學研究所考試(工程數學試題)

1. A differential equation: $(ax+by)dx+(kx+ly)dy=0$, here a, b, k and l are constants.
(10 point)
(a) Under what condition does the above differential equation become an exact differential equation?
(b) Solve the exact differential equation of (a).

2. Find the general solution of the following equation:
(10 point)
$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=\frac{6}{x}$$

3. Find the general solution of the following equation:
(10 point)
$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x\sin x$$

4. Consider a model of undamped mass-spring system:
(10 point)
$$\frac{d^2y}{dx^2}+\omega^2y=\gamma(t), \quad (\omega\neq 0)$$
here $\gamma(t)$ is an unspecified driving force, The initial conditions are $y(0)=K_1$, $\left.\frac{dy}{dt}\right|_{t=0}=K_2$ where K_1 and K_2 are constants.
Use the Laplace Transformation to find $y(t)$ in terms of $\gamma(t)$, K_1 & K_2.
(注意：使用其他方法不給分).

5. Use the method of matrix diagonalization to solve
(10 point)
$$\frac{dy_1}{dt}=5y_1+8y_2+1$$
$$\frac{dy_2}{dt}=-6y_1-9y_2+t$$
with initial conditions: $y_1(0)=4$, $y_2(0)=-3$.
(注意：使用其他方法不給分)

6. (a) What type of graph in X_1-X_2 plane can be represented by the quadratic
(10 point) form : $10X_1^2 - 9X_1X_2 + \frac{25}{4}X_2^2 = 13$

Use the Matrix Theory to discuss your answer !

(b) Is the quadratic form in (a) "Positive definite" ? Why ?

7. Let f and g be continuous functions in some domain $\mathbf{D}$, that contains a region
(10 point) R and its boundary surface S. f and g satisfy the assumptions in the divergence theorem.

(a) Derive the Green's Second Formula for f and g.

(b) Use (a) in the 2 dimensional domain, to evaluate $\oint_C \frac{\partial \omega}{\partial n} ds$ counterclockwisely, where $\omega = x^3 + 3x^2 - 3xy^2$ and the $\vec{n}$ is the outnormal vector of curve C : $(x-4)^2 + (y+2)^2 = 10$.

8. (a) Find the Fourier Series of $f(x) = |x|$, $-\pi < x < \pi$ with period 2π.
(10 point) (b) Using the Bessel Inequality to show :

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \cdots = \frac{\pi^4}{96}$$

9. Consider a set of function $\{y_1, y_2, \cdots, y_m, y_n, \cdots\}$ is the solution of the Sturm-
(10 point) Liouville problem $[r(x)y']' + [q(x) + \lambda P(x)]y = 0$, corresponding to the eigenvalue $\{\lambda_1, \lambda_2, \cdots, \lambda_m, \lambda_n, \cdots\}$. Prove (y_m, y_n) are orthogonal with respect to $P(x)$, if $a \le x \le b$, $r(a) \ne 0$, $r(b) \ne 0$ and the boundary conditions are

$$k_1 y(a) + k_2 y'(a) = 0$$
$$l_1 y(b) + l_2 y'(b) = 0$$

here k_1, k_2, l_1 & l_2 are constants, and (k_1, k_2) and (l_1, l_2) are not zero at the same time.

10. Use the Residual Theorem of the "Theory of complex variables" to
(10 point) evaluate

$$\int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} dx.$$