1. The diffusion in the melt must be described by the more general equation

$$D_L \nabla^2 C_L + V_I \frac{dC_L}{dx} = 0$$

Assume the following physically realistic conditions:

(i) $C_L \rightarrow C_E$ as $x \rightarrow \infty$ (the melt remains of eutectic composition far from the solid/liquid interface).

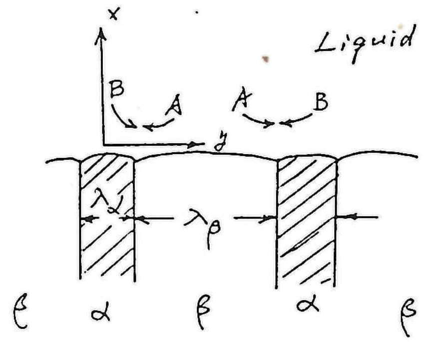
(II) $dC_L/dy = 0$ above the centers of the lamellae (i.e. at $y = 0$ and $y = 1/2(\lambda_\alpha + \lambda_\beta)$). and

(iii) At steady state the diffusive flux of A and B atoms through the liquid onto the interface leads to the formation of α and β at the same rate V_I:

$$\left(\frac{dC_L}{dx}\right)_{x=0} = -\frac{V_I C_\alpha}{D_L} \quad \text{for } -1/2\lambda_\alpha \leq y \leq 1/2\lambda_\alpha$$

$$\left(\frac{dC_L}{dx}\right)_{x=0} = -\frac{V_I C_\beta}{D_L} \quad \text{for } 1/2\lambda_\alpha \leq y \leq 1/2\lambda_\alpha + \lambda_\beta$$

Solve C_L. (20%)



2. Using the method of determinants to solve the system:

$$x' = -2x + y$$
$$y' = -3x + 2y + 2\sin t$$

(15%)

3. Let S be the closed parabolic bowl consisting of the two pieces: S_1: $z = x^2 + y^2$; $x^2 + y^2 \leq 1$ and

S_2: $x = r\cos\theta$, $y = r\sin$,

$z = 1$; $0 \leq r \leq 1$, $0 \leq \theta \leq 2\pi$.

Let $F = (x - y + z)i + 2xj + k$, prove the Gauss's formula.
(20%)

4. Find the Fourier integral representation of the function (10%)

$$f(x) = \begin{cases} 1 & \text{when } x < 1, \\ 0 & \text{when } x > 1. \end{cases}$$

5. Using the Laplace transformation to solve: (15%)

$y'' - 4y' + 3y = 0$, $y(0) = 3$, $Y'(0) = 7$.

6. Is the field given by $F = 2xyzi + (e^y + x^2z)j + x^2yk$ conservative? If it is, find a potential.
(20%)