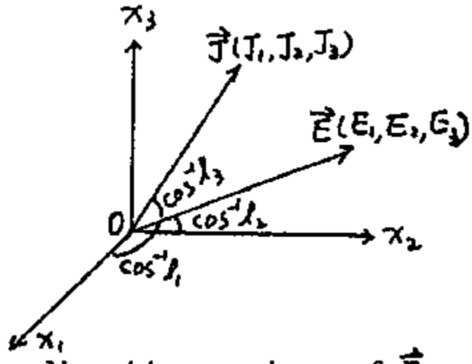
**** You must solve the problems in consecutive order. ****

1. Select either (a) or (b) to solve: (15 points)

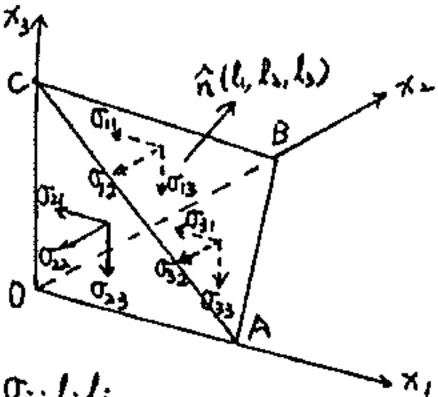
(a) As shown in the right figure where Ox_1, Ox_2 and Ox_3 are three orthogonal axes, the current density $\vec{J}(J_1, J_2, J_3)$ is generally not parallel to the applied electric field $\vec{E}(E_1, E_2, E_3)$ for an anisotropic material. According to Ohm's law,

$$J_i = \sum_{j=1}^{3} \sigma_{ij} E_j, \quad i = 1, 2 \text{ or } 3,$$

where σ_{ij} are material constants (electrical conductivities). If the direction cosines of $\vec{E}$ relative to Ox_1, Ox_2 and Ox_3 are l_1, l_2 and l_3, show that the current density parallel to $\vec{E}$ is $\sum_{i,j=1}^{3} \sigma_{ij} l_i l_j$, where E is the magnitude of $\vec{E}$.



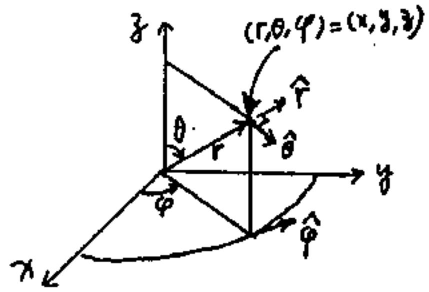
(b) Consider force equlibrium of an infinitesimal tetrahedron OABC of which OA, OB and OC are along the orthogonal axes Ox_1, Ox_2 and Ox_3. The unit vector along the outward normal of the oblique plane ABC is $\hat{n}(l_1, l_2, l_3)$. The stresses σ_{ij} acting on the three planes containing the edges OA, OB and OC are specified in the right figure. Show that the normal stress acting on ABC is $\sum_{i,j=1}^{3} \sigma_{ij} l_i l_j$.



2. Prove that the gradient of a scalar function, ψ, is given by

$$\nabla \psi(r, \theta, \varphi) = \frac{\partial \psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial \psi}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial \varphi}\hat{\varphi}$$

in a spherical polar coordinate system, (r, θ, φ), shown in the right figure where $\hat{r}$, $\hat{\theta}$ and $\hat{\varphi}$ are the three unit vectors. (15 points)



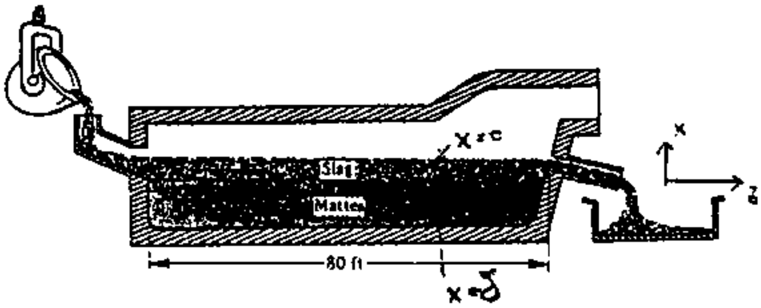
3. Evaluate the integral

$$I = \int_{-\infty}^{\infty} \frac{\cos x}{\pi^2 - 4x^2}\, dx. \qquad \text{(20 points)}$$

4. Molten slag is passed over a matte in the smelting of copper in order to recover most of the copper in the slag. The operation is carried out in a reverberatory furnace 80 ft long and 30 ft wide as shown in the figure below. Assuming that the matte is stationary and that the slag flows continuously (80 ft^3/hr) over the matte. It is known that the flow behavior is governed by the following equation

$$\frac{d\tau_{xz}}{dx} = \rho g \cos\beta \text{ , and } \tau_{xz} = -\eta \frac{dV_z}{dx}$$

where V_z is the velocity of the slag. Please determine (a) the equation for the velocity distribution in the slag layer; by knowing the boundary conditions that at $x=0$, $\tau_{xz}=0$ and at $x=\delta$, $V_z=0$, (b) the fraction of slag which flows in a velocity slower than half the mean velocity. The average depth of the slag may be taken as 2 ft. It should be known that $\bar{V_z} = \int_0^\delta V_z\, dx$ and $Q=\bar{V_z} W \delta$ where $\bar{V_z}$ is the mean velocity. (20 points)



5. Consider a nuclear fuel element of spherical form as shown below. It consists of a sphere of fissionable material with radius $R^{(F)}$, surrounded by a spherical shell of aluminum cladding with outer radius $R^{(C)}$. Inside the fuel element fission fragments are produced which have very high kinetic energies. Collisions between the fragments and the atoms of the fissionable material provide the major source of the thermal energy in the reactor. Such a volume source of thermal energy resulting from nuclear fusion we call Sn (cal/cm^{3}sec). This source will not be uniform throughout the sphere of fissionable material; it will be the smallest at the cell of the sphere. It is known that the source can be approximated by a simple parabolic function:

$$S_n = S_{no}\left[1 + b\left(\frac{r}{R^{(F)}}\right)^2\right]$$

Here Sno is the volume rate of heat production at the center of the sphere and b is a dimensionless constant between 0 and 1. Assume that the governing differential equations for the heat fluxes in the fissionable sphere and the aluminum cladding are as follows:

$$\frac{d}{dr}\left(r^2 q_r^{(F)}\right) = S_{no}\, r^2\left[1 + b\left(\frac{r}{R^{(F)}}\right)^2\right]$$

and

$$\frac{d}{dr}\left(r^2 q_r^{(C)}\right) = 0$$

The boundary conditions are as follows:

at $r=0$, $q_r^{(F)}$ is not infinite, at $r=R^{(F)}$, $q_r^{(F)} = q_r^{(C)}$

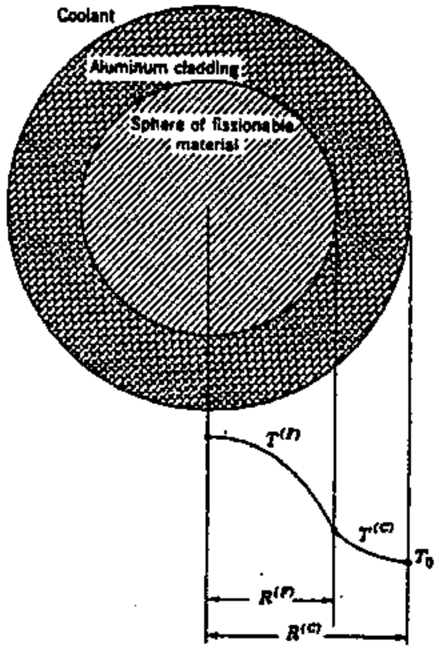
at $r=R^{(F)}$, $T^{(F)}=T^{(C)}$, at $r=R^{(C)}$, $T^{(C)} = T_0$

It is also known from Fourier law of heat conduction that

$$q_r^{(F)} = -k^{(F)} \frac{dT^{(F)}}{dr}$$

$$q_r^{(C)} = -k^{(C)} \frac{dT^{(C)}}{dr}$$

Please find the temperature profiles in the fissionable sphere and aluminum cladding. (15 points)



Temperature distribution in a spherical nuclear fuel assembly.

6. Find the Fourier transform of e^{-px^2}, where p>0. (15 points)