1. Let $\mathbf{v} = x(y+1)z^3\boldsymbol{j}$, and let V be the pentahedron with faces given by the planes $x = 0$, $x = 2$, $y = 0$, $z = 0$, and $y + x = 1$. Evaluate the two sides of the equation of Divergence theorem and to verify that they are equal.

2. Consider the conduction of heat in a rod, the lateral surface of which is not insulated. If heat is convected from the rod to the ambient medium, the partial differential equation governing the temperature $u(x,t)$ is

$$\alpha^2\, \partial^2 u/\partial x^2 = u_t + h(u - u_\infty) \qquad \text{(a)}$$

where the constants h and u_∞ are the convective heat transfer coefficient and the ambient temperature, respectively. Our interest in (a) lies in the Newton cooling term $h(u - u_\infty)$. Although it is not essential, one normally begins by eliminating the u_∞ term by setting $v(x,t) = u(x,t) - u_\infty$ and considering, instead, $\alpha^2\, \partial^2 u/\partial x^2 = u_t + hv$. Solve the Newton cooling problem

$$\alpha^2\, \partial^2 v/\partial x^2 = v_t + hv \quad (0<x<l,\ 0<t<\infty)$$
$$v(0,t) = v(l,t) = 50, \quad v(x,0) = f(x)$$

by separation of variables, leaving expansion coefficients in integral form.

3. Consider the Dirichlet problem shown in the follow

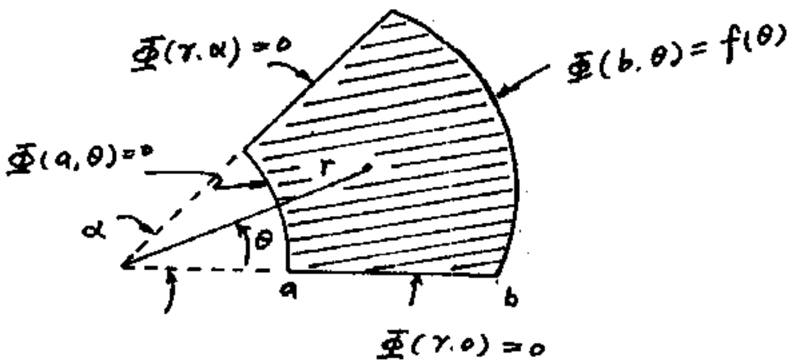
$$\nabla^2\Phi = \partial^2\Phi/\partial r^2 + (1/r)\,\partial\Phi/\partial r + (1/r^2)\,\partial^2\Phi/\partial\theta^2 = 0$$
$$(a<r<b,\ 0<\theta<\alpha)$$

$\Phi(r,0) = 0 \quad (a<r<b)$

$\Phi(r,\alpha) = 0 \quad (a<r<b)$

$\Phi(a,\theta) = 0 \quad (0<\theta<\alpha)$

$\Phi(b,\theta) = f(\theta) \quad (0<\theta<\alpha)$



4. Solve the vibrating rectangular membrane problem

$$c^2(\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2) = \partial^2 w/\partial t^2 \qquad (0<x<a,\ 0<y<b, 0<t<\infty)$$

$$w(0,y,t) = w(a,y,t) = 0$$
$$w(x,0,t) = w(x,b,t) = 0$$
$$w(x,y,0) = f(x,y), \qquad \partial w(x,y,0)/\partial t = 0$$

for $w(x,y,t)$ by separation of variables, leaving expansion coefficients in integral form.

5. Let ρ, ϕ, θ be spherical polar coordinates, and let $u(\rho,t)$ be the spherically symmetric temperature field within a sphere of radius c, where t is the time. Solve the problem

$$\alpha^2\nabla^2 u = \alpha^2(\partial^2 u/\partial\rho^2 + (2/\rho)\partial u/\partial\rho) = \partial u/\partial t \qquad (0\le\rho<c, 0<t<\infty)$$

$$u(\rho,0) = f(\rho),$$
$$u(c,t) = 0$$