

1. A  $10 \times 25 \times 50$ -mm block of 0.40%C hot-rolled steel is placed between two rigid frictionless walls and subjected to the stress shown in Fig. 1. Determine the change in length of the 10-mm side of the block. (For 1040 hot-rolled steel:  $\sigma_y = 260$  MPa,  $\sigma_{UTS} = 580$  MPa, Elongation in 50 mm = 29%,  $E = 210$  GPa,  $G = 80$  GPa) [20%]
2. Consider the Mohr's circle for 3-D stress transformation.  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  represent respectively, the three normal stress components along the three principal axes (by convention,  $\sigma_1 \geq \sigma_2 \geq \sigma_3$  algebraically) so that a graphical representation of the Mohr's circle is illustrated in Fig. 2, where all possible stress states lie within the shaded area. (a) What is the maximum shear stress ( $\tau_{max}$ ) in this case? [5%] (b) Consider the von Mises yield criterion, prove that the plastic flow in a solid is caused by shear stresses. [10%] (c) Explain why a hydrostatic component of stress has no influence on the plastic-flow criteria. [10%]
3. The torque tube in Fig. 3, with outer diameter  $d_o = 1.25$  in. and inner diameter 1.00 in., is subjected to a torque  $T = 1000$  lb in. A strain gage oriented at an angle  $\theta = -45^\circ$  with respect to the axis, which measures the extensional strain along this direction, gives a reading of  $\epsilon_{-45} = 190 \mu\text{in./in.}$  [10%] (a) Determine the value of the maximum shear stress,  $\tau_{max}$ . (b) Determine the shear modulus of elasticity,  $G$ . [15%] [Hint: Moment of inertia  $I$  for circular bars =  $\frac{1}{2}\pi r^4$ ]
4. Consider an aluminum thin film (thickness =  $d_f$ ) sputter-deposited onto a Si wafer substrate with diameter = 200 mm and thickness =  $d_s$ . Suppose now the Si wafer is bent by a moment during processing (as shown in Fig. 4) so that force and moment are induced in the film and wafer:  $F_f$  and  $M_f$  in the film,  $F_s$  and  $M_s$  in the Si substrate, where  $F_f = F_s$ . In this case, the deformation is assumed to consist entirely of the extension or contraction of longitudinal beam fibers by an amount proportional to their distance from the neutral axis, which remains unstrained in the process. (a) Show that the stress in the film  $\sigma_f = \frac{1}{6R} \frac{E_s d_s^2}{(1-\nu_s) d_f}$ , where  $E_s$  and  $\nu_s$  are the elastic modulus, and Poisson's ratio, respectively, for the Si substrate. [14%] [Hint: Assuming bi-axial stress condition.] (b) After deposition, the wafer (Al+Si) is heated up to  $400^\circ\text{C}$  and then cooled down gradually to room temperature. Derive an expression for the thermal stress in the Al film. In addition, draw schematically a stress-temperature curve for the Al film during this thermal process. Explain your drawing. [10%] (c) The Al film is patterned into isolated  $1\text{-}\mu\text{m}$  wide conductor lines on the Si substrate. Compare the magnitude and state of the stresses in Al in this case with those in case (a). [6%]

