(1) (8%) Find the limit if it exists.

(a) $\lim_{x\to 0}(1-x)^{\frac{1}{x}}$.

(b) $\lim_{t\to 0}\int_t^{2t}\frac{e^{-x}}{x}$.

(2) Determine whether the series is convergent or divergent. Justify your answer

(a) (4%) $\sum_{m=0}^{\infty}\frac{2m+3}{(m+1)2^m}$.

(b) (4%) $\sum_{k=2}^{\infty}\frac{(-1)^{k-1}}{\ln k}$.

(c) (6%) $\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi}{2}}{n^p}$, for any $(p>0)$.

Hint: You may find the fact that $\sin(k\pi+\frac{\pi}{2})=(-1)^{k+1}$ useful.

(3) (6%) Determine the radius of convergence of the following power series

$$\sum_{n=1}^{\infty}\frac{n^n}{n!}x^n.$$

(4) (12%) Let

$$F(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0).\end{cases}$$

(a) Find the values $F_x(0,0)$ and $F_y(0,0)$ such that F_x and F_y are continuous in an open disc containing $(0,0)$.

Hint: You may find the following formulas $F_x=\frac{y(x^4-y^4)+4x^2y^3}{(x^2+y^2)^2}$ and $F_y=\frac{x(x^4-y^4)-4x^3y^2}{(x^2+y^2)^2}$ for $(x,y)\neq(0,0)$ useful.

(b) Show that $F_{xy}(0,0)\neq F_{yx}(0,0)$.

(5) (6%) Find the extrema of the function

$$F(x,y)=x^3-12x+y^2$$

for $x,y\in(-\infty,\infty)$.

(6) (30%) Evaluate each of the following integrals.

(a) $\int e^x \sin 2x \, dx$.

(b) $\int \frac{1}{x^{1/2}+x^{1/3}} \, dx$.

(c) $\int_C (x^2 y dx + y^3 dy)$, where C is the arc of the parabola $x = y^2$ from $(0,0)$ to $(1,1)$.

(d) $\int_{-\infty}^{\infty} e^{-x^2} \, dx$.

(e) $\int_0^1 \int_{y=2x}^{y=2} \frac{\sin y}{y} \, dy \, dx$.

(7) (6%) Find the area of the region bounded by the graphs of $y = x^2 - 4$ and $y = 4 - x^2$.

(8) (6%) Find the volume of the solid obtained by rotating the region bounded by $y = \frac{1}{x}$, $x = 1$, $x = 3$, and $y = 0$ about the x axis.

(9) (12%) Determine whether the following is **True** or **False**. Justify your answer.

(a) If $f(x)$ is continuous at $x = c$, then $f(x)$ is differentiable at $x = c$.

(b) If there is a number B such that $\frac{|f(x)-L|}{|x-c|} \leq B$ for all $x \neq c$, then $\lim_{x \to c} f(x) = L$.