

1. Find the limits.

(i) $\lim_{n \rightarrow \infty} (\sqrt{n^2 - n + 1} - n)$ (5%)

(ii) $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$ (5%)

(iii) $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y}{x^2 + y^2}$ (5%)

(iv) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{n^2 + k}}$ (5%)

2. Evaluate the integrals.

(i) $\int_1^4 e^{\sqrt{x}} dx$ (10%)

(ii) $\int_0^1 \int_y^1 \frac{1}{1+x^4} dx dy$ (10%)

3. Find $\frac{d^2}{dx^2} \int_0^x \int_1^{\sin t} \sqrt{1+u^4} du dt$. (10%)

4. Find an equation of the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$. (10%)

5. Find the highest and lowest points on the curve $x^2 + xy + y^2 = 12$. (14%)

6. Find the work done by the force field $F(x, y) = (e^x - y^3, x^3 + \cos y)$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counterclockwise direction. (14%)

7. Let $f(x, y, z) = \frac{e^{x+y+z}}{xyz}$ for $x > 0, y > 0$ and $z > 0$. Show that f has the minimum value e^3 . (12%)