注意： (1) 嚴禁使用任何電子輔助工具作答。 (2) 請依題號先後順序作答。

1. $y_i = f(x_i)$ is the measured datum of $f(x)$ at $x = x_i$, where $i = 0, 1, 2$ or 3 (i.e., $y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$ and $y_3 = f(x_3)$). Taking $f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ for data fitting, find the coefficients of the polynomial associated with the experimental results listed in the following table. (15%)

Variable, x	Measured Datum, y
$x = x_0 = 0$	$y = y_0 = 10$
$x = x_1 = 2$	$y = y_1 = 22$
$x = x_2 = 4$	$y = y_2 = 38$
$x = x_3 = 6$	$y = y_3 = 50$

2. (a) Draw the curve of $x^2 + 4y^2 = 4$ in x-y Cartesian coordinate system. (2%)

   (b) Consider another Cartesian coordinate system, x'-y', of common origin with x-y. Rotating in counter-clockwise sense, x'-axis is 30^O to y-axis and y'-axis is 30^O to x-axis. In terms of x' and y', derive the equation of the curve in Problem (a). (12%)

3. Solve the equation $x^{2x+5} = x^{11}$. (12%)

4. Mathematical modeling of diffusion in isotropic solids is concerned in this problem. Consider a volume element in the form of rectangular parallelepiped whose sides are parallel to the axes of Cartesian coordinate system and are of lengths $2\times\delta x$, $2\times\delta y$ and $2\times\delta z$. Let the center of the element be at P(x, y, x), where the concentration of diffusing substance is C. Let ABCD be the face of the element at $x + \delta x$. Then the rate at which diffusing substance leaves the element through ABCD is $4 \times \delta y \times \delta z \times [J_x + (\partial J_x/\partial x)\,\delta x]$, where J_x is the rate of transfer of diffusing substance through unit area of the corresponding plane through P. Using the above information and others, derive the equation that $\partial C/\partial t = -(\partial J_x/\partial x + \partial J_y/\partial y + \partial J_z/\partial z)$, where $\partial C/\partial t$ is the time derivative of C, and J_y and J_z have the same meaning of J_x but through the planes perpendicular to y-axis and z-axis, respectively. (背面仍有題目,請繼續作答) (15%)

5. Solve the equation $dx/(xz) = dy/(yz) = -dz/(x^2 + y^2)$. (15%)

6. What is the condition for the following set of homogeneous linear equations of x, y and z to satisfy $x^2 + y^2 + z^2 \neq 0$? Why?

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

(14%)

7. A solid of fixed composition is subjected to temperature change (dT) and volume change (dV) in a reversible thermodynamic process. The corresponding change of Helmholtz free energy (dF) can be expressed as

$$dF = -S\,dT - P\,dV,$$

where S and P are entropy and pressure, respectively. Derive the equation

$$(\partial S/\partial V)_T = (\partial P/\partial T)_V.$$

Emphasize the mathematical concept in your derivation. (15%)