系所組別： 電機工程學系,微電子工程研究所,電腦與通信工程研究所甲丁組

考試科目： 工程數學

考試日期：0226，節次：3

※ 考生請注意：本試題可使用計算機，並限「考選部核定之國家考試電子計算器」機型

1. (15%) Use separation of variables to find the general solution. Then, obtain the particular satisfying the given initial condition. Sketch the graph of the solution, showing the key features, and label any key values

$$y' = (y^2 - y)e^x; \qquad y(0) = 2$$

2. (20%) To solve $x^2y'' - 2xy' + 4y = 0$

3. (20%) If $f(z)$ is analytic for $|z| < 1$ and $|f(z)| \le \frac{1}{1-|z|}$

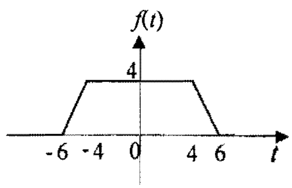
Find the best estimated value of $f^{(n)}(0)$ that Cauchy's formula will yield, express it in terms of n . (Hint: Use Cauchy's formula in each $|z| < r$, $(r < 1)$).

4. (15%) Find the smallest positive integers *m* and *n* such that

$$(\sqrt{3} - i)^m = (1 + i)^n$$

5. (15%) For the following figure, the Fourier Transform of $f(t)$ is denoted as $\hat{f}(\omega)$.

(A). Find $\hat{f}(\omega)$. (B). Evaluate $\int_{-\infty}^{\infty} \left|\hat{f}(\omega)\right|^2 d\omega$



6. (15%) Consider the problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{for } 0 < x < a, \quad 0 < y < b$$

$$\frac{\partial u}{\partial y}(x,0) = f(x) \quad \text{for } 0 \le x \le a$$

$$\frac{\partial u}{\partial y}(x,b) = g(x) \quad \text{for } 0 \le x \le a$$

$$\frac{\partial u}{\partial x}(0,y) = \frac{\partial u}{\partial x}(a,y) = 0 \quad \text{for } 0 \le y \le b$$

(A). State the necessary condition for existence of a solution.

(B). Solve the problem for $a=b=\pi$, $f(x)=\cos(2x)\cos(x)-\sin(2x)\sin(x)$, $g(x) = 3x - \frac{3}{2}\pi$.