

系所組別：電機工程學系丁組、電腦與通信工程研究所甲組

考試科目：資料結構

考試日期：0222，節次：2

※ 考生請注意：(務必詳讀，以免影響權益。)

- (1) 本試題不可使用計算機。
- (2) 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。
- (3) 本試題共計 4 頁。作答時可不必抄題，但請務必於答案卷將各作答題之題號標註清楚。

Note that, throughout this paper, the height of a tree with only one node is defined as 1 while an empty tree has height of zero.

1. 是非題 [A] (7 分；下列 (1-1)~(1-7) 各題敘述若正確請以 T 或 O 表示，若錯誤請以 F 或 × 表示；每題 1 分，答對得 1 分，答錯或未作答得 0 分)

For each statement in (1-1) ~ (1-7), please indicate T or O if it is correct and indicate F or × otherwise. (1 points each)

- (1-1) If an undirected connected graph G has no bridge edge then G is a strongly connected graph.
- (1-2) A B-tree of order 2 is an AVL tree.
- (1-3) The best case time complexity of a comparison-based sorting algorithm can achieve $O(n)$.
- (1-4) A double-ended priority queue supports the operation of deleting an element with arbitrary priority.
- (1-5) The worst case performance of searching using a hash table is the same as using a binary search tree.
- (1-6) A min tree is also a winner tree.
- (1-7) Top-down splay trees, bottom-up splay trees, red-black trees, B^+ -trees are all binary search trees.

2. 是非題 [B] (12 分；下列 (2-1)~(2-6) 各題敘述若正確請以 T 或 O 表示，若錯誤請以 F 或 × 表示；每題 2 分，答對得 2 分，答錯得 0 分並倒扣 1 分，未作答得 0 分)

For each statement in (2-1) ~ (2-6), please indicate T or O if it is correct and indicate F or × otherwise. (You earn 2 points for each correct answer and lose 1 point for each incorrect answer.)

- (2-1) A graph G is bipartite if all cycles in G are of even length.
- (2-2) Heap sort algorithm can be implemented as a recursive algorithm which uses linked list(s).
- (2-3) For an undirected connected graph $G = (V, E)$, where V is the set of vertices and E is the set of edges. If G is a multigraph, then $|V|(|V|-1)/2 \leq |E| \leq |V|^2$ holds.
- (2-4) For static hashing with linear open addressing to be efficient, the loading factor $\alpha > 1.0$ must hold.
- (2-5) In terms of space complexity, selection sort is always more efficient than quick sort.
- (2-6) If T is an AVL tree and the balance factor of its root is 0, then T becomes imbalanced right after inserting a single node.

3. 名詞或術語解釋 (21 分；請解釋下列 (3-1)~(3-7)各題之名詞或術語；每題 3 分)

Term explanation (3 points each)

- | | | |
|--------------------------------|----------------------------|--------------------------|
| (3-1) Stable sorting algorithm | (3-2) Amortized complexity | (3-3) LRb imbalance |
| (3-4) Biconnected component | (3-5) Bloom filter | (3-6) Articulation point |
| (3-7) Circular queue | | |

(背面仍有題目，請繼續作答)

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4. 選擇題 (10 分，每小題 2 分)

For each question from (4-1) to (4-5), please choose the most suitable item from the ones given in the box below as your answer. Note that all the items in the box can be chosen for more than one question. In case that you think none of the items in the box can be used as answer, mark \emptyset as your answer. (2 points each)

(A) $O(1)$	(B) $O(n)$	(C) $O(n^2)$	(D) $O(n^m)$
(E) $O(n \cdot \log n)$	(F) $O(\log n)$	(G) $O(n^2 \cdot \log n)$	(H) $O(2^n)$

Questions (4-1) ~ (4-5) refer to the following program code:

```
#define N n
#define M x
#include <stdlib.h>
int main(){
    float Data[N], T;
    int I, J, K;
    /* Part A Begins ----- */
    for (I = 0; I < N; I++)
        Data[I] = rand()*N+1;
    K = 1; I = 0, J = N-2;
    while (K <= M) {
        if(Data[I] > Data[I+1]){
            T = Data[I];
            Data[I] = Data[I+1];
            Data[I+1] = T;
        }
        I++; K++;
        if(I > J){I = 0; J--;}
    }
    /* Part A Ends ----- */
    /* Part B Begins ----- */
    

Program code sorting all the elements of
        array Data using sorting algorithm y


    /* Part B Ends ----- */
}
```

- (4-1) What is the time complexity of Part A if the values of n and x satisfy $n = (x/3)$? (4-1)
- (4-2) Suppose that the value x is less than n and the Part B uses *Merge Sort Algorithm* as y to sort the array Data into non-increasing order, then what is the time complexity of Part B? (4-2)
- (4-3) Suppose that the value x is greater than n^2 and the Part B uses *Quick Sort Algorithm* as y to sort the array Data into non-decreasing order, then what is the time complexity of Part B? (4-3)
- (4-4) Suppose that the value x is greater than n^2 and the Part B uses *Bubble Sort Algorithm* as y to sort the array Data into non-decreasing order, then what is the time complexity of Part B? (4-4)
- (4-5) Suppose that the value x is less than n and the Part B uses *Bubble Sort Algorithm* as y to sort the array Data into non-increasing order, then what is the total time complexity of Part A and Part B? (4-5)

5. 選擇題 (10 分，每小題 2 分)

For each question from (5-1) to (5-5), please choose the most suitable item from the ones given. In case that you think none of the given items can be selected as answer, mark \emptyset as your answer. (2 points each)

(5-1) Given a binary search tree and R is its root node, if the balance factor of R 's left child is -1 and the balance factor of R 's right child is 1, then which is the balance factor of R ?

- (A) -2 (B) 2 (C) -1 (D) 1 (E) 0

(5-2) Which value results from valuating the prefix express "+ * * 6 - 5 2 - + 4 2 6 / 8 - * 6 2 8"?

- (A) 0 (B) 2 (C) 4 (D) 3 (E) 1

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(5-3) Suppose that the post-order traversal performed on a binary tree T produces the sequence of nodes "A, B, C, D, E, F, G, H, J, K, L". If the right sub-tree of T 's root node has only 2 nodes, then which is the left child of T 's root node?

- (A) A (B) D (C) F (D) G (E) H (F) J (G) L

(5-4) The time complexity of building an AVL tree of n keys in the worst case is

- (A) $O(n)$ (B) $O(n^2)$ (C) $O(n \cdot \log n)$ (D) $O(n^2 \cdot \log n)$ (E) $O(n^3)$ (F) $O(n \cdot (\log n)^2)$

(5-5) Which traversal operation is used in tree sort?

- (A) Level-order (B) In-order (C) Pre-order (D) Post-order (E) BFS (F) DFS

6. 複擇題 (40 分, 每小題 5 分)

(6-1) Insert a sequence of numbers (66, 8, 37, 41, 56, 39, 72) sequentially into an empty AVL tree T , which of the following statement(s) is (are) true about the resultant T ?

- (A) The height of T is 3.
 (B) The number of leaf nodes is 5.
 (C) T is a full binary tree.
 (D) There are 2 RL rotations in constructing the resultant T .
 (E) There is 1 RR rotation in constructing the resultant T .

(6-2) Suppose an optimal binary search tree T contains (5, 32, 55, 96). T is constructed based on the user search information given in the table below.

Search value	0 - 4	5	6 - 31	32	33 - 54	55	56 - 95	96	97 - 99
Frequency	5%	18%	9%	16%	6%	22%	8%	12%	4%

Which of the following statement(s) is (are) true about T ?

- (A) The height of T is 4.
 (B) 32 is at the root of T .
 (C) T is an AVL tree.
 (D) (32, 5, 96, 55) is the postorder traversal of T .
 (E) The average cost for searching each key in T once is 3.

(6-3) Given a preorder traversal sequence (F, B, H, A, C, E, D, G, I) of a binary tree and a postorder traversal sequence (A, H, B, E, I, G, D, C, F) of a binary tree. Which of the following statement(s) is (are) true?

- (A) The postorder sequence and the preorder sequence uniquely define a binary tree.
 (B) If given an inorder traversal sequence (A, H, E, B, F, G, I, D, C), it and the postorder sequence together uniquely defines a binary tree whose height is 4.
 (C) A tree T that satisfies the given preorder and postorder traversals must have C as the root of its right subtree.
 (D) A tree T that satisfies the given preorder and postorder traversals must have A at level 3.
 (E) A possible binary tree that satisfies the given preorder and postorder traversals can be height of 5.

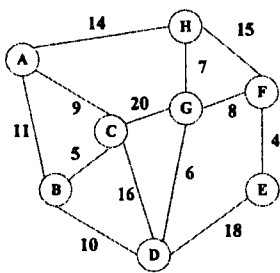
(背面仍有題目, 請繼續作答)

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(6-4) Given an undirected graph $G(V, E)$ where the number of vertexes is n and the number of edges is m . Assume G is represented by adjacency lists. Which of the following statement(s) is (are) true about the graph G ?

- (A) The time complexity to determine whether G is connected is $O(n + m)$.
- (B) The space complexity is $O(m)$.
- (C) The time complexity for determining if a node in G is an articulation point is $O(n^2)$.
- (D) If G is connected, then $n - 1 \leq m \leq n(n-1)$.
- (E) If G is connected, the time complexity to determine a spanning tree of G is $O(n + m)$.

(6-5) Given a weighted undirected graph $G(V, E)$ below. Construct the minimum spanning tree (MST). Which of the following statement(s) is (are) true?

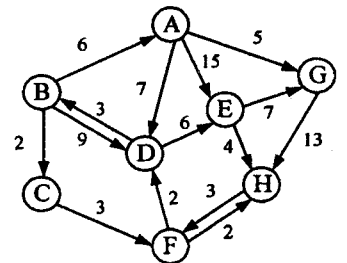


- (A) If the MST is constructed by Kruskal's algorithm, edge (H, F) is the 6th edge added to the MST.
- (B) If the MST is constructed by Kruskal's algorithm, the MST is a binary tree.
- (C) If the MST is constructed by Prim's algorithm and starting from vertex D, edge (B, C) is the 6th edge added to the MST.
- (D) If the MST is constructed by Prim's algorithm and starting from vertex D, vertex A is vertex H's descendant.
- (E) If the MST is constructed by Sollin's algorithm, assuming vertex G as the root, the MST has height of 5.

(6-6) Given a hash function $h(x) = x \% 11$ and a hash table $ht[]$. Assume $ht[]$ has only one slot in each entry and applies open addressing. Now, hash the keys (42, 67, 52, 21, 92, 74, 65, 86, 40, 18) sequentially into $ht[]$. Note that $ht[]$ entry starts at 0. Which of the following statement(s) is (are) true?

- (A) Collisions and overflows occur at the same time in $ht[]$.
- (B) The average number of comparisons for finding each key in $ht[]$ is 3.
- (C) 65 is placed in entry 3 of $ht[]$.
- (D) 74 is placed in entry 1 of $ht[]$.
- (E) The number of comparisons for finding 18 in $ht[]$ is 10.

(6-7) Given the weighted, directed graph below. Using Dijkstra's algorithm, find the shortest path starting from vertex A to all other vertexes. Assume A is the first vertex added to the set S in finding the shortest paths. Which of the following statement(s) is (are) true?



- (A) F is the last vertex added to set S .
- (B) B is the 4th vertex added to set S .
- (C) The shortest path from A to E is 13.
- (D) The shortest path from A to H is 18.
- (E) If weight of the edge from C to F is changed to -8, Dijkstra's algorithm is still able to find the shortest path.

(6-8) Given a sequence of numbers (35, 54, 12, 63, 27}. Suppose a sorting algorithm S is applied to sort the numbers into ascending order. If (12, 63, 54, 35, 27) is a temperate order during the sorting process. Which of the following statement(s) is (are) true?

- (A) S can be quick sort.
- (B) S can be merge sort.
- (C) S can be radix sort.
- (D) S can be insertion sort.
- (E) The time complexity of S is $O(n \log n)$.