※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15 %) Solve the boundary value problem.
   $y'' - 9y = 0$, $y(-4) = y(4) = \cosh 12$

2. (20%) Solve the initial value problem
   $Y''' - y'' - 4y' + 4y = 6e^{-x}$, $y(0)=2$, $y'(0)=3$, $y''(0)=-1$

3. (5%) (a)What kind of singularity (if any) does

   $f(z) = \frac{1}{2z} + \frac{1}{(2z)^2} + \frac{1}{(2z)^3} + \cdots$

   have at $z = 0$

   (10%) (b) Let $f(z) = \frac{1}{z^2(z+2i)}$

   Find a Laurent series in powers of z which converges to f in region on $0 < |z| < 2$

4. (20%) Evaluate $I = \int_0^\infty \frac{x^{1/3}}{(x+1)^2}dx$

5. (10%) Let f be continuous on $[-L, L]$ and let f' be piecewise continuous. Suppose $f(-L) = f(L)$. Prove that the Fourier series of f uniformly and absolutely converges to f.

6. (20%) Consider a circular membrance. Using polar coordinates, the particle of membrane at (r, θ) is assumed to vibrate vertical to the x, y plane, and its displacement from the rest position at time tis $z(r, \theta, t)$. The wave equation for this displacement function can be expressed as

$$\frac{\partial^2 z}{\partial t^2} = c^2\left(\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2}\right)$$

   Where c is a constant. Assume that:

   甲、The rest position of the membrane be in the x, y plane with origin at the center with radius R.

   乙、The motion of the membrane is symmetric about origin, in which case z depends only on r and t, i.e. $z(r, \theta, t) = z(r, t)$ and $\partial^2 z/\partial\theta^2 = 0$.

   丙、It is fastened onto a circular frame, i.e. $z(R, t) = 0$.

   丁、It is set in motion with given initial position $z(r, 0) = f(r)$ and velocity $\partial z(r, 0)/\partial t = g(r)$.

   Solve this boundary value problem, i.e. find $z(r, t)$.