

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (17%) Solve

$$\frac{\partial^2 z}{\partial t^2} = 4 \left( \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right)$$

for  $0 < x < 2\pi, 0 < y < 2\pi, t > 0$  with following initial and boundary conditions.

$$z(x, 0, t) = z(x, 2\pi, t) = 0 \quad \text{for } 0 < x < 2\pi, t > 0,$$

$$z(0, y, t) = z(2\pi, y, t) = 0 \quad \text{for } 0 < y < 2\pi, t > 0,$$

$$z(x, y, 0) = 0 \quad \text{for } 0 < x < 2\pi, 0 < y < 2\pi,$$

$$\frac{\partial z}{\partial t}(x, y, 0) = 1 \quad \text{for } 0 < x < 2\pi, 0 < y < 2\pi.$$

2. (15%) Consider Bessel function  $J_\nu(x)$  of order  $\nu \geq 0$  with  $j_n$  as its  $n$ -th zero. Prove that the functions  $\sqrt{x}J_\nu(j_n x)$ , for  $n = 1, 2, 3, \dots$ , are orthogonal on  $[0, 1]$  in the sense that

$$\int_0^1 x J_\nu(j_n x) J_\nu(j_m x) dx = 0 \quad \text{if } n \neq m.$$

3. (15%) Find a general solution of the following differential equation  
 $y'' - 2y' + y = e^x \sin x$

4. (18%) Find the following initial value problem

$$y^{IV} + 3y'' - 4y = 0$$

$$y(0)=0, y'(0)=-20, y''(0)=0, y'''(0)=80$$

5. Evaluate the Cauchy principal value of

(a)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx$  (10%)

(b)  $\int_0^{\infty} \frac{x \sin x}{x^2+9} dx$  (10%)

6. Use contour integration to evaluate

$$\int_0^{\infty} \frac{(\ln x)^2}{x^2+1} dx \quad (15\%)$$