

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the following differential equation by using the Laplace transform method: (25%)

$$\frac{d^2x(t)}{dt^2} + 8\frac{dx(t)}{dt} + 25x(t) = \sin 3t; \quad x(0) = 1; \quad \frac{dx}{dt}(0) = 2$$

2. Find the state-space representation in parallel form for the following system: (25%)

$$G(s) = \frac{-250(s^2 + 2s + 80)}{(s - 1.6)(s + 4.5)(s + 12)}$$

3. (a) Use Routh (or Routh-Hurwitz) stability criterion to determine how many roots of the equation, $s^5 + 8s^4 + 9s^3 - 18s^2 + 20s = 200$, are located in the left half plane, on the $j\omega$ axis, and in the right half plane. (12%)

(b) If there are some roots on the $j\omega$ axis, please determine them. (8%)

4. Consider the LTI system with state equation $\dot{\mathbf{x}}_{n \times 1}(t) = \mathbf{A}_{n \times n} \mathbf{x}(t) + \mathbf{B}_{n \times p} \mathbf{u}_{p \times 1}$, initial $\mathbf{x}(0) = \mathbf{x}_0$, and output equation $\mathbf{y}(t)_{q \times 1} = \mathbf{C}_{q \times n} \mathbf{x}(t) + \mathbf{D}_{q \times p} \mathbf{u}(t)$. Please derive and find the impulse response matrix $\mathbf{G}_{q \times p}(t)$. (12%)

5. **Definition:** An initially relaxed system is said to be bounded-input bounded-output (BIBO) stable if every bounded input excites a bounded output.

Definition: A function $h(t)$ is said to be absolutely integrable in $(0, \infty)$ if $\int_0^\infty |h(t)| dt \leq m < \infty$ for some positive constant m .

Consider a single-input single-output (SISO) linear time-invariant system,

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = \int_0^t g(\tau)u(t-\tau)d\tau \quad (\text{Equation } \alpha)$$

where $y(t)$ is the output, $g(t)$ is the impulse response, and $u(t)$ is the bounded input with $|u(t)| \leq u_m < \infty$ for all $t \geq 0$. Please show that a SISO system described by Equation (α) is BIBO stable if and only if $g(t)$ is absolutely integrable in $(0, \infty)$. (18%)