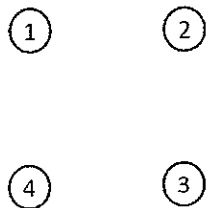


※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

- 1 (5 pts) Which of the following statement(s) is (are) true about a binary search tree T with n nodes?
- (A) A binary tree is a binary search tree if both the left subtree and the right subtree of the root are binary search tree.
 - (B) The time complexity of inserting a key into T is $O(\log n)$
 - (C) The number of nodes with two children is less than the number of leaf nodes.
 - (D) The time complexity of deleting a key in T can be performed in $O(h)$ if the search tree has a height of h .
 - (E) None of above.
2. (5 pts) Which of the following statement(s) is (are) true?
- (A) Dijkstra's algorithm can be used to find the single source single destination shortest path for two nodes in any graph.
 - (B) Bellman-Ford algorithm can be used to check if a graph contains a negative-weight cycle.
 - (C) Prim's algorithm is a greedy algorithm.
 - (D) Kruskal's algorithm requires to use disjoint-set data structure to check if an edge will form a cycle while finding a minimal spanning tree.
 - (E) None of above.
3. (5 pts) A depth-first algorithm classifies the edges of a graph into tree, back, forward, and cross edges. If we classify the edges reachable from the source of a breadth-first tree into the same four categories. Please show which of the following properties are true after applying breath-first search algorithm to a **directed graph**.
- (A) There are no cross edges.
 - (B) There are no forward edges.
 - (C) For each tree edge (u, v) , we have $d[v]=d[u]+1$.
 - (D) For each cross edge (u, v) , we have $d[v] \leq d[u]+1$
 - (E) For each back edge (u, v) , we have $0 \leq d[v] < d[u]$
4. (15 pts) Binary search tree.
- (1) (6 pts) Please show a binary tree with HJBFGECA of postorder traversal and HBJAFDGCE of inorder traversal (step by step).
 - (2) (3 pts) Draw the threaded representation of the binary tree in (1).
 - (3) (6 pts) Please show the resulting figure after inserting I as left child of C.
5. (15 pts) The Floyd-Warshall Algorithm find shortest paths between each pair of nodes. Suppose we label all nodes in the graph from 1 to n . Let $A^k(i, j)$ denote an element in a matrix which represent the shortest path from i to j through nodes $1 \sim k$.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

(1) (2 pts) Please show the initial graph according to the sequence of the following figure.



(2) (5 pts) Please show the general recursive equation for generating $A^k(i, j)$.

(3) (6 pts) Please show $A^1(i, j)$ and $A^2(i, j)$.

(4) (2 pts) How many iteration does it need to perform in order to obtain the correct result?

6. (10 pts) Given two single linked lists list1 and list2, where the values in them are ordered in the ascending order.

```

struct item {
    int value;
    struct item *next;
}
int func(struct item *list1, struct item *list2) {
    Struct item *ptr1, * ptr2;
    int count = 0;
    ptr1 = list1;
    ptr2 = list2;
    while ((ptr1 !=NULL) && (ptr2 !=NULL)) {
        if ((ptr1->value < ptr2->value) ptr1 = ptr1->next;
        if ((ptr1->value < ptr2->value) ptr1 = ptr1->next;
        else {
            ptr1= ptr1->Next;
            ptr2= ptr2->Next;
            count++;
        }
    }
    Return count;
}

```

(1). (4 pts) What is the value returned by `func(list1, list2)`, where `list1` and `list2` are given below.

`list1`-> 5 ->7->13->24->59->99->100

`list2`-> 1->10->24->25->99->120

(2) (3 pts) What function does `func(list1, list2)` perform?

(3) (3 pts) What is the worst-case time complexity of function(`list1, list2`) if `m` and `n` are the numbers of items in `list 1` and `list2`.

7. (20 pts) Given an array `A` which contains `n` elements. `A` is used to represent a complete tree.

MAX-HEAPIFY(`A, i`)

```

1.  $l \leftarrow \text{LEFT}(i)$ 
2.  $r \leftarrow \text{RIGHT}(i)$ 
3. if  $l \leq \text{heap-size}[A]$  and  $A[l] > A[i]$ 
4.   then  $\text{largest} \leftarrow l$ 
5.   else  $\text{largest} \leftarrow i$ 
6. if  $r \leq \text{heap-size}[A]$  and  $A[r] > A[\text{largest}]$ 
7.   then  $\text{largest} \leftarrow r$ 
8. if  $\text{largest} \neq i$ 
9.   then exchange  $A[i] \leftrightarrow A[\text{largest}]$ 
10.   MAX-HEAPIFY( $A, \text{largest}$ )

```

HEAPSORT(`A`)

```

1. BUILD-MAX-HEAP( $A$ )
2. for  $i \leftarrow \text{length}[A]$  downto 2
3.   do exchange  $A[1] \leftrightarrow A[i]$ 
4.    $\text{heap-size}[A] \leftarrow \text{heap-size}[A] - 1$ 
5.   MAX-HEAPIFY( $A, 1$ )

```

(1) (3 pts) Please give the definition of max heap shortly.

(2) (3 pts) Please explain the function of MAX-HEAPIFY shortly.

(3) (3 pts) Please show a simple code BUILD-MAX_HEAP to build a max heap based on MAX-HEAPIFY. Suppose an array contains `n` elements. In MAX-HEAPIFY, `l` and `r` denote the left and right child of node `i`, respectively, and `heap-size[A]` denotes number of elements stored in `A`.

(4) (3 pts) What is the time complexity of your algorithm?

(5) (4 pts) Please show the result after applying your algorithm BUILD-MAX_HEAP on `A = <5, 3, 17, 10, 84 >`.

(6) (4 pts) Please show the resulting graph after apply HEAPSORT on your graph (step by step).

8. (15 pts) We use the forest representation to record disjoint-sets. There are two strategies which can reduce runtime of the operations, which include weighting rule for Union(`i, j`) and collapsing rule for Find(`i`). Initially, there exist six sets, `S(0)`, `S(1)`, `S(2)`, `S(3)`, `S(4)`, `S(5)`, and `S(6)`, where each set contains only one element.

(1) (6 pts) Please explain weighting rule for Union(`i, j`) and collapsing rule for Find(`i`) shortly.

(2) (6 pts) Please show the resulting graph after performing Union(`0, 1`), Union(`0, 3`), Union(`2, 4`), Union(`5, 6`), Union(`2, 5`), Union(`0, 2`) (step by step).

(3) (3 pts) Please show the resulting graph after performing Find(`6`).

9. (10 pts) Please answer the following problems shortly.

(1) (5 pts) Please show one strategy to judge if a circular queue is full.

(2) (5 pts) Please show one strategy to improve running time of Quicksort algorithm.