編號： 182

國立成功大學 105 學年度碩士班招生考試試題

系　　所：電機工程學系

考試科目：離散數學

考試日期：0227，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

請按題號作答

1. (15%) Design one circuit for both three-bit (x,y,z) even parity generator and four-bit even parity checker.

2. (15%) Prove that "$\equiv \bmod n$" relation on $\mathbf{Z}$ is an equivalence relation.

3. (15%) What is the following recursive definition defining. λ means the empty string.
   (a) $\lambda^R=\lambda$; If s has one or more symbols, write s=ra where a is a symbol and r is a string. Then $s^R=(ra)^R=ar^R$.

   (b) λis in X; If x is in X, so are 1x0 and 0x1; If x and y are in X, so is xy.

4. (15%) Using the nouns N={dog, man , mouse, bird} and the verbs V{bites, eats , kicks}, how many "sentences" of the form " <noun><verb><noun>" are there, with the restriction that every word in the sentence has a different length? Use a decision tree to arrive at your answer.

5. (10%) In a class of 36, there will always be a group of at least n persons who were born on the same day of the week. What is n?

6. (10%) In the US, the most commonly used coins come in values of 1, 5, 10, 25. Consider the problem of forming N cents using pennies (1 ¢), nickels (5 ¢), dimes (10 ¢), and quarters (25 ¢). A greedy algorithm for making change is as follows:

```
Preconditions: N, w<x<y<z
Postconditions: w*p+x*n+y*d+z*q=N, and p+n+d+q is as small as possible.
Pseducode:
p, n, d, q ← 0
T← p+5n+10d+25q
while T<N do
 if (N-T)≧z then
       q←q+1
 else
       if (N-T) ≧y then
             d←d+1
       else
             if (N-T)≧x then
                   n←n+1
             else
                   p←p+1
       T←w*p+x*n+y*d+z*q
```

However, it is not obvious that this algorithm always gives an optimal amount of change when w, x, y, z are free value. Give an example of the values w, x, y, z, and N, and **discuss how the above algorithm fails to generate an optimal solution.**

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7. (10%) What does the following algorithm generates when n=6, r=4?

```
Input: r, n
test(r,n) {
  for i=1 to r
        s_i=i
  print(s_1,...,s_r)
  for i =2 to C(n,r) {
        m=r
        max_val=n
        while (sm == max_val) {
              m=m-1
              max_val=max_val-1
        }
        s_m=s_m+1
        for j=m+1 to r
              s_j=s_{j-1} +1
        print(s_1, ..., s_r)
  }
}
```

8. (10%) Use induction to show that if $r \neq 1$, $a+ar+ar^2+\ldots+ar^n=\frac{a(r^{n+1}-1)}{r-1}$.