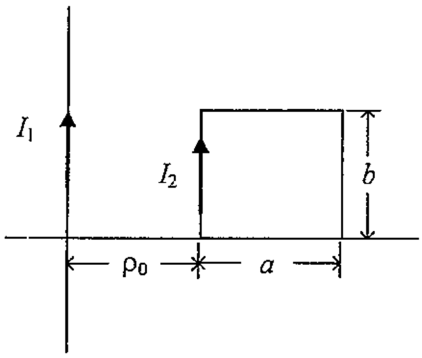
編號： 176　　　　**國立成功大學 106 學年度碩士班招生考試試題**

系　所：電機工程學系

考試科目：電磁學　　　　考試日期：0213，節次：2

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

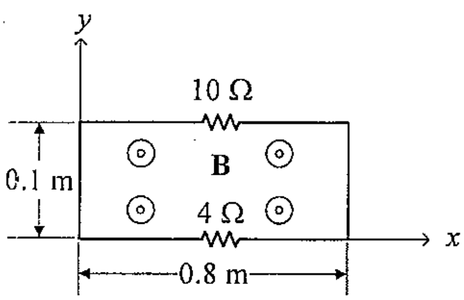
1. [10 points] Find the expression for the mutual inductance between the rectangular loop and the infinite line current as shown below.



2. [20 points] The volume charge density inside a sphere of radius a is $\rho_v = \rho_0\left(1-\dfrac{r^2}{a^2}\right)$, where ρ_0 is a constant.

   (a) Calculate the total charge.
   (b) Determine electric intensity field $\mathbf{E}$ and potential V outside the sphere.
   (c) Determine electric intensity field $\mathbf{E}$ and potential V inside the sphere.
   (d) Find the maximum of $\mathbf{E}$ and the corresponding position.

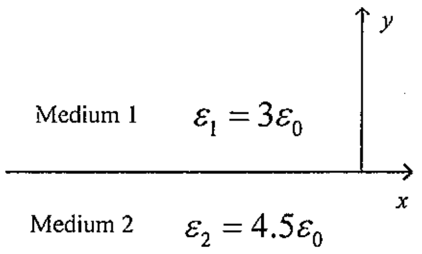
3. [10 points] The circuit shown below exists in a magnetic field $\mathbf{B} = 40\cos(30\pi t - 3y)\mathbf{a}_z$ mWb/m^2. Assume that the wire connecting the resistors have negligible resistances. Find the current in the circuit.



4. [10 points] An antenna radiates in free space and the magnetic field is
$\mathbf{H} = \dfrac{12\sin\theta}{r}\cos\left(2\pi\times10^8 t - \beta r\right)\mathbf{a}_\theta$ mA/m . Find the corresponding $\mathbf{E}$ in terms of β.

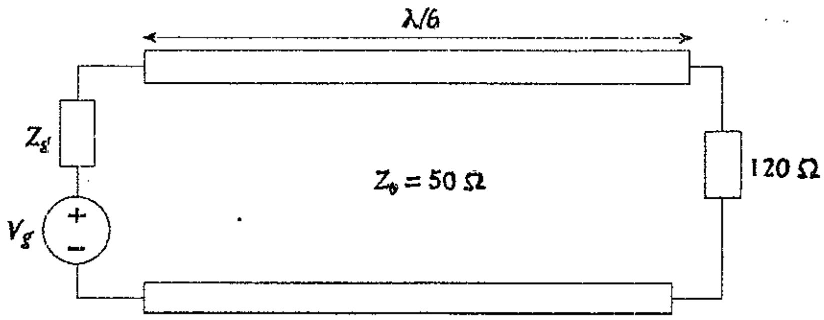
5. [10 points] A grounded metal sheet is located in the $z = 0$ plane, while a point charge Q is located at $(0, 0, a)$. Find the force acting on a point charge $-Q$ placed at $(a, 0, a)$.

6. [20 points] Given that $\mathbf{E}_1 = 10\mathbf{a}_x - 6\mathbf{a}_y + 12\mathbf{a}_z$ V/m in the figure shown below, find (a) polarization vector $\mathbf{P}_1$ in medium 1, (b) electric field $\mathbf{E}_2$ in medium 2 and the angle $\mathbf{E}_2$ makes with the y-axis, (c) the energy density in each region.



7. [10 points] Refer to the lossless transmission line shown below. (a) Find reflection coefficient Γ_L and VSWR.

   (b) Determine input impedance Z_{in} at the generator.



8. [10 points] The dimension a of a parallel-plate waveguide with a dielectric of $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$ is 3 cm. Determine the propagating modes for a wave of frequency 6000 MHz.

Some formula for your reference:

$$\varepsilon_0 = \frac{10^{-9}}{36\pi}\ \text{F/m},\quad \mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$$

In cylindrical coordinates:

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z,\quad \nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix},\quad \nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

In spherical coordinates:

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi,\quad \nabla \cdot \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho D_\rho\right) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & r\sin\theta\,\mathbf{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$