編號： 182

國立成功大學 106 學年度碩士班招生考試試題

系　　所：電機工程學系

考試科目：離散數學

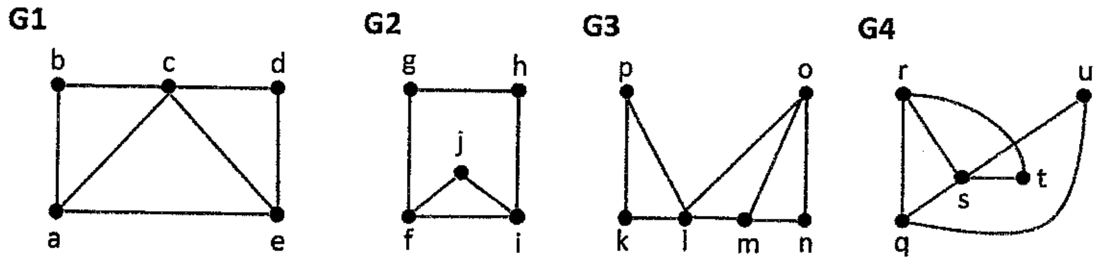
考試日期：0213，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) A function f: Z→Z is called parity preserving if f(n) is even when n is even, and f(n) is odd when n is odd. Give an example of a parity-preserving function (besides the identity). Derive an inductive proof to verify it.

2. (10%) (a) Which pair of graphs below is isomorphic?

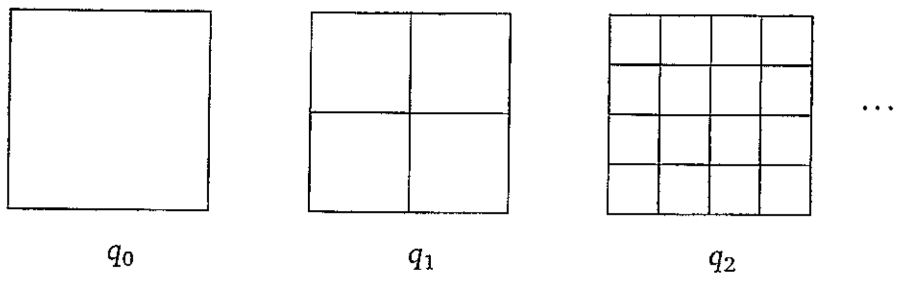
   (b) Give the one-to-one correspondence: $V_G \rightarrow V_H$ of vertices for this pair of graph.



將右表畫於答案卷上作答。

V	α(V)

3. (10%) The figure below shows the first three squares in a sequence $q_0, q_1, q_2, \ldots$ of squares divided into square regions. **Give a <u>recursive definition</u> for the above sequence of shapes.**



4. (10%) Alice, Bob, Cindy are trying to decide who should get the last doughnut. Alice suggests that they flip two coins: if both are heads, Bob gets it. If both are tails, Cindy gets it. In other cases, Alice gets it. If you are Bob or Cindy, will you accept this suggestion? If not, give a fair suggestion.

5. (10%) Claude and Maude are playing a game with poker chips. The object of the game is to draw the last chip. The players take turns drawing chips. At each turn, a player must take at least one chip and at most ten chips. Claude goes first. What is an optimal strategy? Does the strategy depend on the number of chips left?

6. (10%)One version of *Ackermann*'s function A (m, n) is defined recursively for m, n ∈ **N** by

   A (0, n) =n+1, n ≥0;

   A (m, 0) =A (m-1, 1), m>0; and

   A (m, n) = A (m-1, A (m, n-1)), m, n>0.

   (a) Calculate A (1, 3) and A (2, 3).

   (b) Prove that A (1, n) = n+2 for all n ∈ **N**.

7. (20%) In computer system, it usually sends messages by encoding a letter with a fixed-length of bits, such as ASCII code. However, to avoid wasting space, a variable-length code is proposed for effectively encoding source symbols. For example, we can encode more frequently occurring letters such as *e* and *a*, with smaller bit strings; and less frequently occurring letters such as *q* and *x* with longer bit strings. This is called the **Huffman coding** method. The **Huffman code** is a particular type of optimal prefix code that is commonly used for lossless data compression. Take the message **BACADAEAFABBAAAGAH** as an example. If a fixed-length encoding table is as below:

A: 000	B: 001	C: 010	D: 011	E: 100	F: 101	G: 110	H: 111

Then the original message will be encoded as a string of 54 bits:

001000010000011000100000101000001001000000000110000111

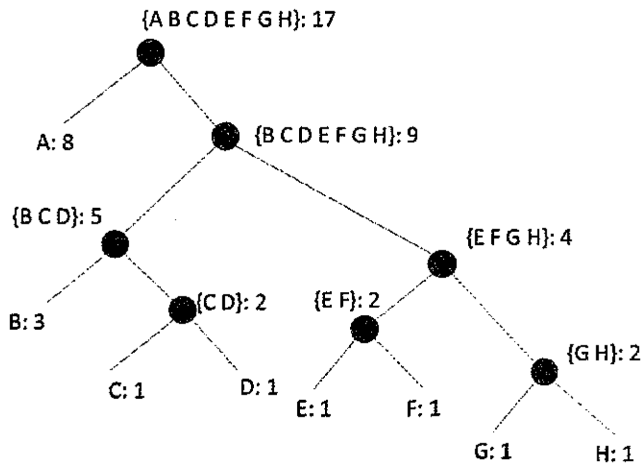
Now considering the frequencies of letters in the message, we can build a Huffman Encoding table as below:

A: 0	B: 100	C: 1010	D: 1011	E: 1100	F: 1101	G: 1110	H: 1111

Based on the Huffman Encoding table, the original message will be encoded as a string of 42 bits:

100010100101101100011010100100000111001111

Here we show the corresponding Huffman encoding tree:



Please answer the following questions:

(a) Use Huffman encoding method to encode the message: (ignore space, comma, dot, and upper/lower case)

Discrete is fun

Show your encoding table, process (including Huffman encoding tree), and the output of encoded message.

(b) Based on the result of (a), what is the average bits of such encoded message?

8. (10%)Use generating functions to solve the following recurrence relation:

$a_n = 5a_{n-1} - 6a_{n-2}$ for n ≥ 2

$a_0 = 0$

$a_1 = 3$

9. (10%)Let w_n denote the number of all possible words of length n, on the alphabet {O, T, Z}, which do not contain the substrings "OT" and "TO". (Note: $w_0 = 1$)

(a) Prove that the numbers w_n satisfy the recurrence relation

$$w_{n+1} = 2w_n + w_{n-1} \,, \quad n = 2, 3, \ldots$$

(b) Based on (a), please find an explicit formula for w_n