

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please derive the peak time  $T_p$  and the percent overshoot %OS for the step response of an underdamped

second-order system,  $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ , in terms of  $\zeta$  and  $\omega_n$ . (25%)

2. For the system represented in state space as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = [2 \quad 3] \mathbf{x}$$

find the diagonal system that is similar. (25%)

3. Consider a closed-loop system described in Figure 1, where  $C(s) = K$  and  $G(s) = \frac{s+1}{s^3 + \alpha s^2 + 5s + 1}$ .

Please apply Routh-Hurwitz method to determine the value of  $K$  and  $\alpha$  so that the system oscillates at a frequency of 2 rad/sec and no poles in the right half of the s-plane. (15%)

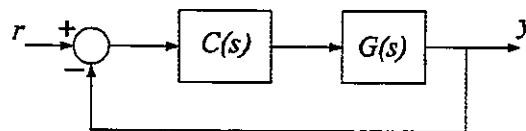


Figure 1

4. For a standard second-order system,  $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ , please derive the bandwidth  $\omega_{BW}$  of  $G(s)$  in terms of  $\zeta$  and  $\omega_n$ . (15%)

5. Consider the following MIMO system,

$$\dot{\mathbf{x}} = \begin{bmatrix} 5 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 & 0 & 1 & 1 \\ 1 & 2 & 2 & 3 & 0 & 2 & 0 \end{bmatrix} \mathbf{x}$$

Please determine the controllability and observability of the eigenvalues 5's and 8's, respectively. (20%)