編號： 174

國立成功大學 108 學年度碩士班招生考試試題

系　　所：電機工程學系

考試科目：離散數學

考試日期：0223，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let A = {0, 1} and, for **a** = (a_1, a_2, a_3) and **b** = (b_1, b_2, b_3) in A^3, define
   a $\preccurlyeq$ **b** if and only if $a_i \le b_i$ for all $1 \le i \le 3$. (20%)
   (a) Prove that (A^3, $\preccurlyeq$) is a partially ordered set. (10%)
   (b) Draw the Hasse diagram of the partial order. (10%)

2. Determine true or false for the following statements and explain your answer. If your answer is "false", you should give a counterexample to point out where is the misstatement. (20%)
   (a) If a relation is both symmetric and transitive, then it is reflexive. (10%)
   (b) If R and S are equivalence relations on a set X, then R$\cup$S is also an equivalence relation on X. (10%)

3. How many integers from 100 through 999 must you pick in order to be sure that at least two of them have a digit in common?(for example, 256 and 530 have the common digit 5)? (10%)

4. Show the generating function to determine the number of ways to insert tokens worth \$1, \$2, and \$5 into a vending machine to pay for an item that costs r dollars in the case then the order in which the tokens are inserted does NOT matter (For example, there are two ways to pay for an item that costs \$3 when the order in which the tokens are inserted does not matter: inserting three \$1 tokens or one \$1 token and a \$2 token. When the order matters, there are three ways: inserting three \$1 tokens, inserting a \$1 token and then a \$2 token, or inserting a \$2 token and then a \$1 token.) (10%)

5. A straight line split a plane into 2 regions, let R_n be the maximum number of region split by n straight lines. (20%)
   (a) What are R_1, R_2, R_3, R_4? (4%)
   (b) Give a recurrence equation for R_n. (6%)
   (c) Find the closed form for R_n. (10%)

6. One urn contains 10 red balls and 25 green balls, and a second urn contains 22 red balls and 15 green balls. A ball is chosen as follows: First an urn is selected by tossing a loaded coin with probability 0.4 of landing heads up and probability 0.6 of landing tails up. If the coin lands heads up, the first urn is chosen; otherwise, the second urn is chosen. Then a ball is picked at random from the chosen urn. (10%)
   (a) What is the probability that the chosen ball is green? (5%)
   (b) If the chosen ball is green, what is the probability that it was picked from the first urn? (5%)

7. A store sells 8 kinds of balloons. If the store has only 12 red balloons and only 8 blue balloons but at least 30 of each other kind of balloon, how many combinations of 30 balloons can be chosen? (10%)