※ 考生請注意：本試題不可使用計算機。　 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. **Solve** $4y'' + 4(e^x - 1)y' + e^{2x}y = 0$ **(15%)**

2. **Prove the integral as following**

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \ , \mathbf{a} > \mathbf{0} \qquad \text{(15%)}$$

3. *(a) Find the eigenvalues ($\lambda_1 \geq \lambda_2 \geq \lambda_3$) of the matrix* $A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **(10%)**

   (b) Obtain an orthogonal matrix P which gives $P^T A P = diag\,(\lambda_1, \lambda_2, \lambda_3)$ **(10%)**

4. **Find the eigenvalues and eigenfunctions of the following problem. (20%)**
   $$y'' + \lambda y = 0; \ y'(0) = y'(L) = 0$$

5. **Determine the inverse Laplace transform of the function** $F(s) = \dfrac{1}{s^2 + 16} e^{-2s}$. **(15%)**

6. **Let vector field** $\mathbf{F} = 2yz\mathbf{i} - 4xz\mathbf{j} + xy\mathbf{k}$. **Evaluate the value of the surface integral** $\iint_{\Sigma} \mathbf{F} \cdot \mathbf{n}\,d\sigma$ **, where** Σ **is the sphere of radius 5 about** $(-1, 3, 1)$. **(15%)**