國立成功大學 109 學年度碩士班招生考試試題

系　　所：電機工程學系

考試科目：電磁學

考試日期：0210，節次：2

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

每題 10 分

1. The Maxwell's equations in differential form are

$$\nabla \cdot \vec{D} = \rho_v \qquad \text{(Gauss' law for electric field)}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \text{(Ampere's law)}$$

(a) Present the procedure to transform Gauss' law for electric field and Ampere's law in terms of scalar potential and vector potential, that is,

$$\nabla^2 V + \frac{\partial}{\partial t}\left(\nabla \cdot \vec{A}\right) = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 \vec{A} - \nabla\left(\nabla \cdot \vec{A}\right) = -\mu \vec{J} + \mu\varepsilon \nabla\left(\frac{\partial V}{\partial t}\right) + \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

(b) Explain why the scalar potential and vector potential can also be found by solving the following equations:

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J}$$

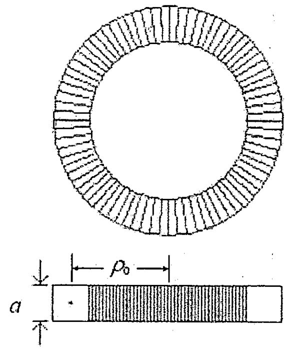
2. An infinite sheet of uniform charge density ρ_s is situated coincident with the xy plane at $z = 0$. The sheet has a hole of radius a centered at the origin. (a) Show that the electric potential V is $-\frac{\rho_s}{2\varepsilon_0}\left(\sqrt{a^2+z^2} - a\right)$ and (b) Find the electric field $\vec{E}$ at points along the z-axis.

3. A conducting sphere of radius a has a total charge Q uniformly distributed on its surface. If the sphere is embedded in a medium with permittivity variation as $\varepsilon = \varepsilon_0\left(1+\frac{a}{r}\right)^2$, find (a) the energy stored and (b) polarization vector.

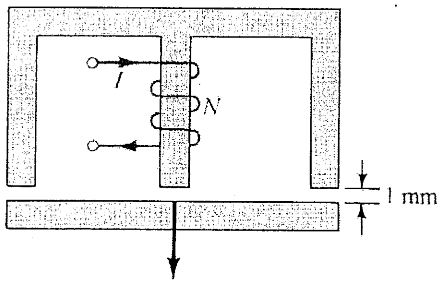
4. A square toroid is shown as the following figure. The length of the side of the square cross section is a. Show that the self-inductance of the toroid is

$$L = \frac{\mu_0 N^2 a}{2\pi} \ln\left[\frac{2\rho_0 + a}{2\rho_0 - a}\right]$$

where N is the number of turns and ρ_0 is the mean radius of the toroid.



5. A section of an electromagnet with a plate below it carrying a load is shown in the figure below. The electromagnet has a contact area of 200 cm^2 per pole, and the middle pole has a winding of 1000 turns with $I = 3$A. Calculate the maximum mass that can be lifted. Assume that the reluctance of the electromagnet and the plate is negligible.



6. A magnetic field is given in the xz-plane by $\vec{B} = B_0 \cos \pi (x - v_0 t)\hat{a}_y$ Wb/m^2. Consider a rigid square loop situated in the xz-plane with its vertices at $(x, 0, 1)$, $(x, 0, 2)$, $(x+1, 0, 2)$ and $(x+1, 0, 1)$. What would be the induced emf if the loop is moving with the velocity $\vec{v} = v_0 \hat{a}_x$ m/s?

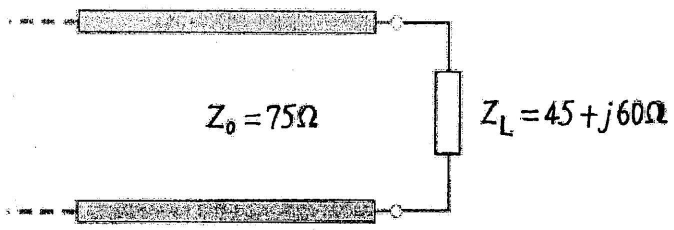
7. The xy-plane serves as the interface between two different media. Media 1 ($z < 0$) is filled with a material whose relative permeability is $\mu_{r1} = 6$, and medium 2 ($z > 0$) is filled with a material whose relative permeability is $\mu_{r2} = 4$. If the material carries surface current density $\vec{K} = (1/\mu_0)\hat{a}_y$ mA/m on the interface, and magnetic flux density field $\vec{B}_2 = 5\hat{a}_x + 8\hat{a}_z$ mWb/m^2 in media 2, find the magnetic flux density field and magnetic intensity field in media 1.

8. A uniform plane wave propagating in a medium has

$$\vec{E} = 2e^{-\alpha z}\sin\left(10^8 t - \beta z\right)\hat{a}_y \text{ V/m}$$

If the medium is characterized by permittivity $\varepsilon = \dfrac{10^{-9}}{36\pi}$ F/m and permeability $\mu = 20\times 4\pi\times 10^{-7}$ H/m, and conductivity $\sigma = 3$ S/m, find α, β, and magnetic intensity field.

9. A 75 Ω lossless transmission line is terminated with an inductive load given by (45 + j60) Ω, as shown below. Calculate (a) the load reflection coefficient Γ_L, (b) the standing-wave ratio SWR on the line, (c) the percentage time-average incident power that is absorbed by the load, and (d) the line voltages V_{max} and V_{min} positions that are nearest to the load.



10. A metallic cavity resonator filled with air is built from a section of a waveguide with dimension $a = 4.755$ cm and $b = 2.215$ cm by shorting the waveguide with two conducting plates with length d along z-axis. The cavity is required to resonate at 8 GHz in the TE_{101} mode. (a) Find the length d of the shorted section necessary. (b) Give the name (TE_{mnl} or TM_{mnl}, looking in the z direction) of the dominant mode (the one with the lowest resonant frequency) and what is its resonant frequency?