國立成功大學

110學年度碩士班招生考試試題

編　號： 174

系　所： 電機工程學系

科　目： 線性代數

日　期： 0202

節　次： 第 3 節

備　註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (24 pts) **Make** each statement **True** or **False** and **JUSTIFY** each answer. 3 pts for each question. (Correct answer for 2 pts and suitable justification for 1 pts.)
   (a). If an $m\times n$ matrix $\boldsymbol{A}$ is row equivalent to an echelon matrix $\boldsymbol{U}$ and if $\boldsymbol{U}$ has k nonzero rows, then the dimension of the solution space of $\boldsymbol{Ax} = \boldsymbol{0}$ is m-k.
   (b). If matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ have the same reduced echelon form, the Row $\boldsymbol{A}$ = Row $\boldsymbol{B}$. (Row $\boldsymbol{A}$ means the row space of $\boldsymbol{A}$.)
   (c). If A is $m\times n$ and rank $A = m$, then the linear transformation $\boldsymbol{x} \mapsto \boldsymbol{Ax}$ is one-to-one.
   (d). If $B = \{\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n\}$ and $C = \{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_n\}$ are bases for a vector space V, then the jth column of the change-of-coordinates matrix P from the base B to C is the coordinates vector $[\boldsymbol{c}_j]_B$.
   (e). If a vector $\boldsymbol{y}$ coincides with its orthogonal projection onto a subspace W, then $\boldsymbol{y}$ is in W.
   (f). If W is a subspace of $\boldsymbol{R}^n$, then W and $W^\perp$ have no vectors in common.
   (g). A least-squares solution of $\boldsymbol{Ax} = \boldsymbol{b}$ is the vector $\boldsymbol{A}\hat{\boldsymbol{x}}$ in Col $\boldsymbol{A}$ closest to $\boldsymbol{b}$, so that $\|\boldsymbol{b}-\boldsymbol{A}\hat{\boldsymbol{x}}\| \le \|\boldsymbol{b}-\boldsymbol{Ax}\|$ for all $\boldsymbol{x}$.
   (h). The normal equations for a least-squares solution of $\boldsymbol{Ax} = \boldsymbol{b}$ are given by $\hat{\boldsymbol{x}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}$.

2. (15 %) Let $T: P_2 \to P_3$ be the transformation that maps a polynomial $\boldsymbol{p}(t)$ into the polynomial $(t+3)\boldsymbol{p}(t)$.
   (a). Find the image of $\boldsymbol{p}(t) = 3 - 2t + t^2$. (5 pts)
   (b). Show that T is a linear transformation. (5 pts)
   (c). Find the matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3\}$. (5 pts)

3. (10 %) $A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$, find bases for Nul A and Col A.

4. (10 %) Find a QR factorization of $A = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$.

5. (21 pts) Let A be the matrix given by $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$ with rank $A = 1$.

   a. Compute the A^+ (the pseudoinverse of A, the inverse of reduced singular value decomposition of A). (10 pts)

   b. Find a least-squares solution for $Ax = b$, where $b = [1, 2, 0]^T$. (6 pts)

   c. Find the least-squares error for part (b). (5 pts)

6. (20 pts) Let $A = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}$, $v_1 = \begin{bmatrix} 3/7 \\ 4/7 \end{bmatrix}$, and $x_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$.

   a. Find a basis for R^2 consisting of v_1 and another eigenvector v_2 of A. (8 pts)

   b. Verify that x_0 may be written in the form $x_0 = v_1 + cv_2$. (5 pts)

   c. For $k = 1, 2, \ldots$, define $x_k = A^k x_0$. Compute x_1 and x_2, and find x^k as k approaches to ∞. (7 pts)