國立成功大學
111學年度碩士班招生考試試題

編　號：174

系　所：電機工程學系

科　目：線性代數

日　期：0219

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (24 pts) **Make** each statement **True** or **False** and **JUSTIFY** each answer. 3 pts for each question. (Correct answer for 2 pts and suitable justification for 1 pts.)

   (a). A is $n \times n$ matrix. If $\lambda+5$ is a factor of the characteristic polynomial of A, then 5 is an eigenvalue of A.

   (b). If A is 3×3, with columns $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{a}_3$, then det A equals the volume of the parallelepiped determined by $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{a}_3$.

   (c). If **u** is in the row space and the column space of an $n \times n$ matrix **A**, then $\mathbf{u} = \mathbf{0}$. **A** is $n \times n$ matrix and **A** is diagonalizable if $\mathbf{A} = \mathbf{PDP}^{-1}$ for some matrix **D** and some invertible matrix **P**.

   (d). If **A** is similar to **B**, then $\mathbf{A}^2$ is similar to $\mathbf{B}^2$.

   (e). The best approximation to **y** by elements of a subspace W is given by the vector $\mathbf{y} - \text{proj}_W\,\mathbf{y}$.

   (f). Not every orthogonal set in $\Re^n$ is linearly independent.

   (g). If **z** is orthogonal to $\mathbf{u}_1$ and to $\mathbf{u}_2$ and if $W = \text{Span}\ \{\mathbf{u}_1; \mathbf{u}_2\}$, then **z** must be in $W^{\perp}$.

   (h). If **b** is in the column space of **A**, then every solution of $\mathbf{Ax} = \mathbf{b}$ is a least-squares solution.

2. (a). (5 pts) Compute the following determinants: $\mathbf{A} = \begin{bmatrix} 9 & 1 & 9 & 9 & 9 \\ 9 & 0 & 9 & 9 & 2 \\ 4 & 0 & 0 & 5 & 0 \\ 9 & 0 & 3 & 9 & 0 \\ 6 & 0 & 0 & 7 & 0 \end{bmatrix}$.

   (b). (10 pts) Find $\mathbf{E}^{2022}$ where $\mathbf{E} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$.

3. (15 points) Let $W = \text{Span}\ \{\mathbf{x}_1, \mathbf{x}_2\}$, where $\mathbf{x}_1 = [-1, -2, 0]^T$ and $\mathbf{x}_2 = [-1, 3, -1]^T$.

   (a). (8 points) Use the Gram-Schmidt process to find an orthogonal basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ for W.

   (b). (7 points) Let $\mathbf{y} = [2, -4, 1]^T$. Decompose **y** as $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$, where $\hat{\mathbf{y}}$ is in W and **z** is in $W^{\perp}$.

4. (46 pts) Let $\mathbf{A} = \begin{bmatrix} 7 & 1 \\ 0 & 0 \\ 5 & 5 \end{bmatrix}$. Find

(a) Find the **LU** factorization of **A**. (5 pts)

(b) Find the **QR** factorization of **A**. (5 pts)

(c) Diagonalize $A^T A$. (5 pts)

(d) Find the maximum value of the quadratic form $\boldsymbol{x}^T(A^T A)\boldsymbol{x}$ subject to the constraint $\boldsymbol{x}^T\boldsymbol{x} = 5$ and find a unit vector at which this maximum value is attained. (5 pts)

(e) Find a singular value of decomposition (SVD) of **A**. (5 pts)

(f) Find an orthonormal basis for $(\text{Col } \mathbf{A})^{\perp}$. (5 pts)

(g) Find $\mathbf{A}^{+}$ (reduced SVD of **A**). (6 pts)

(h) Find the least-squares solution for $A\boldsymbol{x} = [1, 0, 1]^T$. (5 pts)

(i) Find the orthogonal projection of $\boldsymbol{b} = [1, 0, 1]^T$ on Col A. (5 pts)