國立成功大學

112學年度碩士班招生考試試題

編　號：177

系　所：電機工程學系

科　目：線性代數

日　期：0206

節　次：第 3 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30 pts) **Make** each statement **True** or **False** and **JUSTIFY** each answer. 3 pts for each question. (Correct answer for 2 pts and suitable justification for 1 pts.)

(a). A is $n \times n$ matrix. The determinant of A is the product of the pivots in any echelon form U of A, multiplied by $(-1)^r$, where r is the number of row interchanges made during row reduction from A to U.

(b). If a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ spans a finite-dimensional vector space V and if T is a set of more than p vectors in V, then T is linearly dependent.

(c). B and C are bases for a vector space V. If $V = \Re^2$, $B = \{\mathbf{b}_1, \mathbf{b}_2\}$, and $C = \{\mathbf{c}_1, \mathbf{c}_2\}$, then row reduction of $[\mathbf{c}_1\ \mathbf{c}_2\ \mathbf{b}_1\ \mathbf{b}_2]$ to $[\mathbf{I}\ \mathbf{P}]$ produces a matrix $\mathbf{P}$ that satisfies $[\mathbf{x}]_B = \mathbf{P}[\mathbf{x}]_C$ for all $\mathbf{x}$ in V.

(d). A is $n \times n$ matrix. A row replacement operation on A does not change the eigenvalues.

(e). A, B, C are $n \times n$ matrices. If B is similar to A and C is similar to A, then B is similar to C.

(f). The Gram-Schmidt process produces from a linearly independent set $\{\mathbf{x}_1, \ldots, \mathbf{x}_p\}$ an orthogonal set $\{\mathbf{v}1, \ldots, \mathbf{v}_p\}$ with the property that for each k, the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ span the same subspace as that spanned by $\mathbf{x}_1, \ldots, \mathbf{x}_k$.

(g). A is an $m \times n$ matrix and b is in $\Re^m$. A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is a list of weights that, when applied to the columns of A, produces the orthogonal projection of $\mathbf{b}$ onto Col A.

(h). If $B = PDP^T$, where $P^T = P^{-1}$ and D is a diagonal matrix, then B is a symmetric matrix.

(i). The maximum value of $Q(\mathbf{x}) = 7x_1^2 + 3x_2^2 - 2x_1x_2$, subject to the constraint $x_1^2 + x_2^2 = 1$, is $5+\sqrt{5}$.

(j). A is an $m \times n$ matrix with a singular value decomposition $A = U\Sigma V^T$, where U is an $m \times m$ orthogonal matrix, Σ is an $m \times n$ "diagonal" matrix with r positive entries and no negative entries, and V is an $n \times n$ orthogonal matrix. If P is an orthogonal $m \times m$ matrix, then PA has the same singular values as A.

2. (20 pts) Let $\mathbb{P}_3$ have the inner product given by evaluation at -3, -1, 1, and 3. Let $p_0(t) = 1$, $p_1(t) = t$, and $p_2(t) = t^2$.

(a). (10 pts) Compute the orthogonal projection of p_2 onto the subspace spanned by p_0 and p_1.

(b). (10 pts) Find a polynomial q that is orthogonal to p_0 and p_1 such that $\{p_0, p_1, q\}$ is an orthogonal basis for span $\{p_0, p_1, p_2\}$. Scale the polynomial q so that its vector of values at (-3, -1, 1, 3) is (1, -1, -1, 1).

3. (15 pts) Find a factorization $\mathbf{A} = \mathbf{QR}$. $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

4. (35 pts, 7 pts each) Let **A** and its reduced echelon form be given as:

$$\mathbf{A} = \begin{bmatrix} -1 & -5 & 3 & 9 \\ -48 & -40 & 24 & 92 \\ 94 & 70 & -42 & -166 \\ -48 & -40 & 24 & 92 \end{bmatrix}, \mathbf{U} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 10 & -3 & -17 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We already know that $\lambda = 4$ is an eigenvalue of **A**, and $\mathbf{u} = [1, 0, 2, 0]^T$ is an eigenvector of **A**.

(a) Find a basis for the eigenspace for $\lambda = 4$.

(b) What is the eigenvalue for the eigenvector **u**?

(c) Notice that the second and fourth rows of **A** are the same. Does that imply we have a certain eigenvalue? Find a basis for its eigenspace. To save you some time, we have included the reduced echelon form of **A**.

(d) What is the characteristic polynomial of **A**?

(e) Show that **A** is diagonalizable by finding an appropriate **P** and **D**.