

# 國立成功大學

## 113學年度碩士班招生考試試題

編 號： 165

系 所： 電機工程學系

科 目： 電磁學

日 期： 0201

節 次： 第 2 節

備 註： 可使用計算機

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

**Problem 1 (15)**

A unit area parallel-plate capacitor is filled with a medium of a constant conductivity  $\sigma_0$  and a permittivity variation of  $\epsilon(x) = \epsilon_0 \exp(x/d)$ , as shown in the Figure P1, where  $d$  is the thickness of the medium. (a) What is the surface charge at  $x = 0$  and  $x = d$ ? Assume that a voltage drop  $V_0$  is applied and the fringing field is neglected. (b) What is the charge stored in the capacitance? (c) Find the resistance and capacitance of the capacitor.

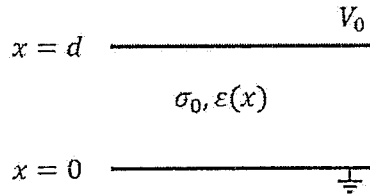


Figure P1

**Problem 2 (15)**

The  $x$ - $z$  plane separates magnetic material with a relative permeability of  $\mu_{r1} = 2.0$  (for  $y < 0$ ) from magnetic material with  $\mu_{r2} = 4.0$  (for  $y > 0$ ). In medium 1, there is a magnetic field  $\vec{H}_1 = 2.0\hat{a}_x + 4.0\hat{a}_y + 6.0\hat{a}_z$  A/m. Find the magnetic field  $\vec{H}_2$  in medium 2, assuming the boundary has a surface current  $\vec{K} = 2.0\hat{a}_x - 2.0\hat{a}_z$  A/m.

**Problem 3 (20)**

A square loop of side  $a$  in free space recedes with a uniform velocity  $u_0\hat{a}_y$  from an infinitely long filament carrying current  $I = I_0 \cos \omega t$  along  $\hat{a}_z$  as shown in the Figure P3. Assuming that  $\rho = \rho_0$  at time  $t = 0$ , find the emf induced in the loop at  $t > 0$ .

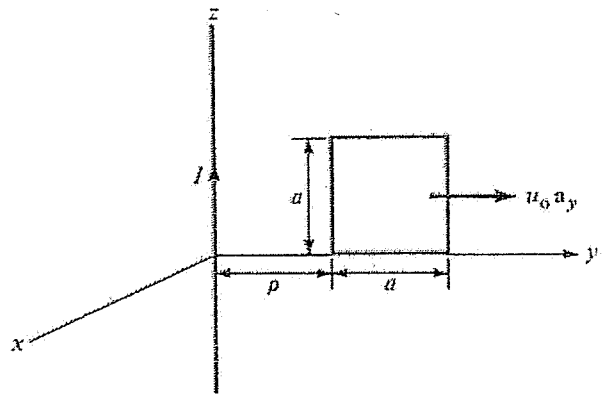


Figure P3

**Problem 4 (20)**

A  $50 \Omega$ , 10 m transmission line (transit time = 36 ns) is fed by a 200 kHz,  $50\sqrt{2} \angle 0^\circ$  V signal generator. A second transmission line is then connected. This  $75 \Omega$ , 2 m transmission line (transit time = 8 ns) follows the first line to transmit signals to a load whose impedance is  $120 - j200 \Omega$ . At the junction of the two transmission lines, calculate (a) the reflection and transmission coefficients, (b) the voltage standing wave ratio (VSWR) of the first line, and (c) the voltage and current.

**Problem 5 (15)**

A parallel-polarized plane wave in free space has

$$\vec{E}_i = (3\hat{a}_x - 4\hat{a}_z) \cos(\omega t - 0.8\pi x - 0.6\pi z)$$

incident on a dielectric surface at the Brewster angle. Assume that the  $xy$ -plane is the interface between the two media.

(a) Find  $\omega$  and the relative permittivity of the dielectric. (b) Determine the magnetic field of the incident wave in free space and (c) the electric field of the transmitted wave in the dielectric.

**Problem 6 (15)**

The general  $z$ -component of the magnetic field for a  $TE_{mn}$  mode in a rectangular waveguide of section  $a \times b$  in phasor form is

$$H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

(a) Prove that the cutoff frequency for the mode is

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

where  $\mu$  and  $\epsilon$  are the permeability and permittivity of the medium filled in the waveguide, respectively.

(b) Prove that the phase constant  $\beta$  for an operating frequency  $f$  can be expressed as

$$\beta = 2\pi f \sqrt{\mu\epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

(c) Prove that the waveguide impedance is given by

$$\eta_{TE} = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

**Some formula for your reference:**

$$\cos(a - bx) \approx \cos a + bx \sin a \text{ for } x \ll 1$$