國立成功大學

113學年度碩士班招生考試試題

編　號：173

系　所：電機工程學系

科　目：離散數學

日　期：0201

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Let S be the set of all strings in 0's and 1's, and define a function F : S →Z as follows: for all strings s in S, F(s) = the number of 1's in s. Is F one-to-one (bijections)? **Yes or No**?

2. (10%) Let A = {0,1} and, for a = (a_1, a_2, a_3) and b = (b_1, b_2, b_3) in A^3, define a ≤ b if and only if a_i ≤ b_i for all 1≦ i ≦3. Is (A^3, ≤) a partially ordered set ? **Yes or No**?

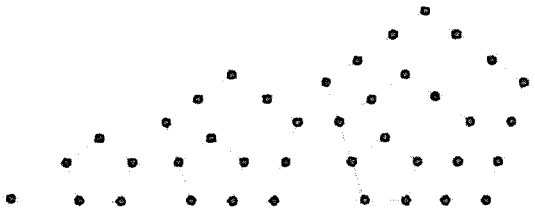
3. (10%) Let T be the set of all two-digit ternary strings; that is, T={00,01,02,10,11,12,20,21,22}. Consider the poset(T, ◁), where ◁ is the bitwise ≦. Let F ⊆ N be the set of all factors of 36, | means divisible. Does (T, ◁) ≅ (F, |) (isomorphic)? **Yes or No**?

4. (10%) Does the following set R define an equivalence relation on the set {1, 2, 3, 4}? Please choose the right answer from (a)(b)(c)(d).
   R = {(1,1), (2,2), (3,3), (4,4), (2,3), (3,2), (2,4), (4,2)}
   (a) No, it's not reflexive.
   (b) No, it's not symmetric.
   (c) No, it's not transitive.
   (d) Yes.

5. (10%) How many of the following examples are second-order linear homogeneous recurrence relations with constant coefficients?
   - $a_k=4a_{k-1}+4a_{k-2}$
   - $a_k=2a_{k-1}+7a_{k-3}$
   - $a_k=3(a_{k-1})^2+2a_{k-2}$
   - $a_k=3a_{k-1}+4a_{k-2}+1$
   - $a_k= -9a_{k-2}$
   - $a_k= 2a_{k-1}$
   - $a_k=4a_{k-1}+(k-1)a_{k-2}$

   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) 5

6. (10%)Early members of the Pythagorean Society defined figurate numbers to be the number of dots in certain geometrical configurations. For example, the first four pentagonal numbers of the following graph are 1, 5, 12, and 22(as figure below). Let P(n) represent the nth pentagonal number, please find **P(n)**.
   (a) P(n) = 1 + 3(2+3+....+n) -2(n-1)
   (b) P(n) = 3(2+3+....+n-2+n-1) -2(n-1)
   (c) P(n) = 1+3(2+3+....+n) -2n
   (d) P(n) = 1+3(2+3+....+n) -2(n-2)
   (e) P(n) = 3(1+2+3+....+n-2+n-1) -2(n)



7. (10%)Find the value of "increment" after the given program segment is executed (Here i, j, k, and increment are integer variables)

```
increments : = 0
for i : = 1 to 22
  for j : = 1 to i
    for k : = 1 to j
        increment : = increment+1
    next k
  next j
next i
```

   (a) 2024
   (b) 1540
   (c) 1771
   (d) 5566
   (e) 7788

8. (10%)Find the "worst-case complexity" of the algorithm segments below.

```
for k:=1 to n-1
  max:=a[k]
  for i:=k+1 to n
    if max<a[i] then max:=a[i]
  next i
a[k]:=max
next k
```

(a) O(n^2)
(b) O(n^3)
(c) O(nlogn)
(d) O(n)
(e) O(logn)

9. (10%)Consider the following algorithm.

i ← 2
while (N mod i) ≠ 0 do
    i ← i + 1

Suppose instead that N ∈ {2,3,4,5,6,7,8}, and all these values are equally likely. Find the "average-case" number of "N mod i" operations made by this algorithm.

(a) 13/7
(b) 16/7
(c) 12/7
(d) 2

10. (10%)How many of the following statements are incorrect?

- C language maintains the recursive steps via stack data structure, which follows the "first in first out" principle.
- When we use recursion to solve problems, we are interested in elegance and efficiency.
- The recursive functions always contain one or more terminating conditions; Without the terminating condition, the recursive function would never stop.
- Recursion is a powerful problem-solving technique that often produces clean solutions to even the most complex problems.

- Every time we make a call, we must use some of the memory resources to make room for the stackframe.

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5