國立成功大學
114學年度碩士班招生考試試題

編　號：121

系　所：電機工程學系

科　目：電磁學

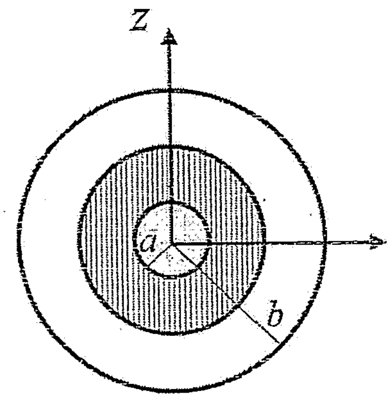
日　期：0210

節　次：第 2 節

注　意：1.可使用計算機
　　　　2.請於答案卷(卡)作答，於試題上作答，不予計分。

Problem 1 (15)

Consider two concentric spherical shells as shown in the figure with the following characteristics: The inner shell has a radius a, carries a total charge of $+Q$ distributed uniformly over its surface. The outer shell has a radius b, carries a total charge of $-Q$ distributed uniformly over its surface. The region between the shells is filled with a dielectric material of permittivity ε for $a < r < (a+b)/2$, while the rest region for $(a+b)/2 < r < b$ is in air. (a) Find the total electrostatic potential energy stored in the system. (b) Find the polarized volume charge density in the dielectric material and the polarized surface charge density at $r = a$ and $r = (a+b)/2$. (c) Determine the electric field induced by the polarized charges for $a < r < b$.

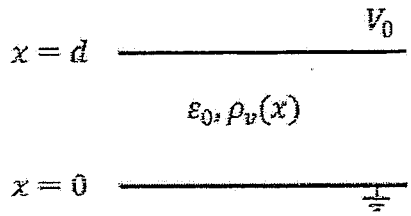


Problem 2 (15)

For the parallel-plate capacitor given in the figure to the right-hand side, suppose a charge density

$$\rho_v = \rho_0 \sin\left(\frac{\pi x}{2d}\right)$$

is distributed between the plates. (a) Derive the expressions for the potential and electric field in the capacitor. (b) Find the capacitance per unit area.

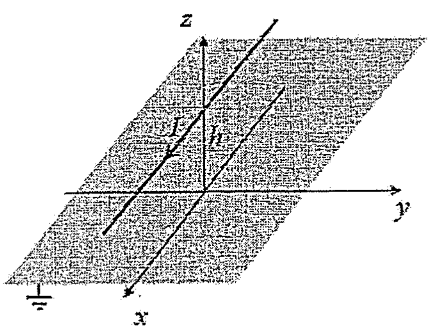


Problem 3 (20)

A current density $\vec{J} = J_z \hat{a}_z$ in a non-magnetic region is distributed such that J_z is a function of ρ for $0 < \rho \leq 1$ in the cylindrical coordinate system, while $J_z = 0$ for $\rho > 1$. Assume the magnetic vector potential is given by $\vec{A} = -\frac{\mu_0}{9}\rho^3 \hat{a}_z$ for $0 < \rho \leq 1$. (a) Find the magnetic energy stored within the volume defined by $0 < \rho \leq 1$, $0 < \phi < 2\pi$ and $0 \leq z \leq 1$. (b) Determine the magnetic vector potential in the region where $\rho > 1$.

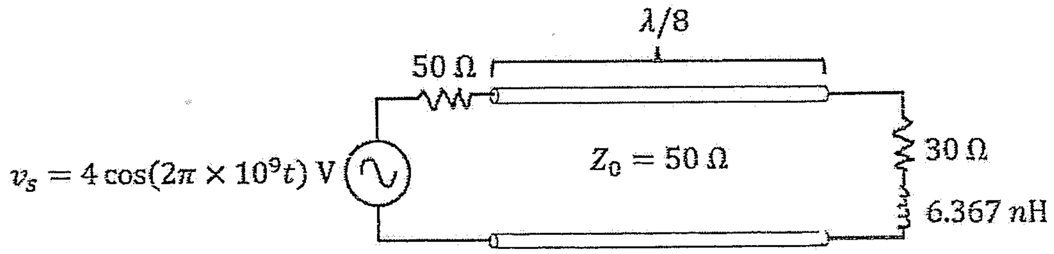
Problem 4 (15)

In the setup shown in the figure, a current I is uniformly distributed in an infinitely long cylinder with radius a, flowing in the positive x-direction. The axis of the cylinder is located at a distance h above an infinitely large, grounded conducting plane in free space, where $a \ll h$.(a) Find the expression for the magnetic field intensity $\vec{H}(x, y, z)$ in the region above the grounded plane and external to the cylinder. (b) Find the total inductance per unit length. (c) Find the surface current density on the grounded plane.



Problem 5 (20)

For the circuit loaded with $30\ \Omega$ and 6.367 nH shown in the figure below, where Z_0 is the characteristic impedance of the transmission line, determine (a) the reflection coefficient at load, (b) the standing wave ratio, (c) the input impedance Z_{in} at the sending end of the transmission line, (d) the instantaneous voltage across the load, and (e) the time-average power and the peak power over the load.



Problem 6 (15)

In a $5\text{ cm} \times 4\text{ cm}$ air-filled rectangular waveguide, the longitudinal field components of a propagation mode at $z = 0$ are given by

$$E_z = 200\sin(40\pi x)\sin(25\pi y)\cos(20\pi \times 10^9 t)\ \text{V/m, and } H_z = 0,$$

where x and y are the spatial coordinates in the transverse directions.

(a) Identify the propagation mode and its cutoff frequency.

(b) Find the wavelength along the propagation direction.

(c) Find the wave impedance of the mode.

Some formula for your reference:

$$\nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\hat{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]\hat{a}_z$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial(A_\phi \sin\theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial(rA_\phi)}{\partial r}\right]\hat{a}_\theta + \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{a}_\phi$$