# 國立成功大學
# 115學年度碩士班招生考試試題

編　號：119

系　所：電機工程學系

科　目：電磁學

日　期：0203

節　次：第 2 節

注　意：1.可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

**Problem 1: 15 points**

Answer the following questions:

(a) How is the polarization vector $\vec{P}$ defined for a medium? Why can $-\nabla \cdot \vec{P}$ be treated as an equivalent bound volume charge density when $\vec{P}$ is non-uniform?

(b) How is the magnetization vector $\vec{M}$ defined for a material? Why can $\nabla \times \vec{M}$ be treated as an equivalent bound volume current density when $\vec{M}$ varies in space?

(c) Ampere's laws are given by

$$\nabla \times \vec{H} = \vec{J} \quad \text{for static fields, and}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{for time-varying fields,}$$

where $\vec{J}$ is a free (conduction) current density. Explain why $\frac{\partial \vec{D}}{\partial t}$ is necessary for time-varying fields.

**Problem 2: 15 points**

A surface charge distribution $\rho_s(x,z)$ exists on the $x-z$ plane, with no charge anywhere else (i.e., $\rho = 0$ for $|y| > 0$). (a) Which of the following potential functions, $\Phi_1$ or $\Phi_2$, is a valid solution for the electrostatic potential in the half-space $y > 0$? (b) What is the potential function for $y < 0$? (c)What is the corresponding charge distribution $\rho_s(x,z)$ on the $xz$ plane?

$$\Phi_1 = e^{-y}\left(\frac{e^{+x} + e^{-x}}{2}\right)$$

$$\Phi_2 = e^{-y}\cos x$$

**Problem 3: 10 points**

An infinite grounded conducting plane occupies the $xy$-plane $(z = 0)$. A real infinite line charge with uniform density $\rho_l$ (C/m) is parallel to the $y$-axis direction and is located at $(x, y, z) = (0,0,b)$. Find the potential $V$ and electric field $\vec{E}$ at point $P = (0,0,b/2)$.

**Problem 4: 15 points**

Two infinitely long coaxial conducting cylinders of radii $a$ and $b$ ( $a < b$ ) form a coaxial structure of length $L$ with $L \gg b$ (ignore end fringing). The region $a < \rho < b$ is filled with a homogeneous medium of permittivity $\varepsilon$ and conductivity $\sigma$. The inner conductor carries free surface charge density $\rho_s$ at $\rho = a$ . The electric flux density in the medium is

$$\vec{D}(\rho) = \frac{\rho_s a}{\rho}\,\hat{a}_\rho, \qquad a < \rho < b,\ 0 < z < L.$$

Model the device between the inner and outer conductors as a capacitor $C$ in parallel with a resistor $R$.

(a) Find $C$ and $R$.

(b) Find the stored electric energy and the power dissipated in the medium (express in terms of $\rho_s$, $a, b, L, \varepsilon, \sigma$).

**Problem 5: 15 points**

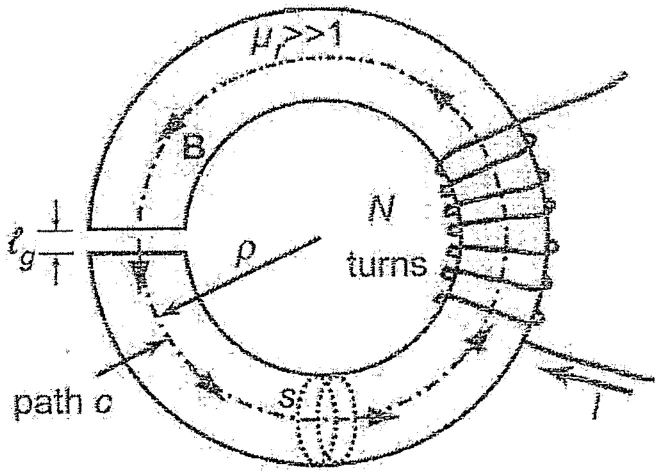
Consider an air-gapped toroidal core with mean radius $\rho$ and uniform cross-sectional area $S$. The core material is linear with relative permeability $\mu_r$ ( $\mu_r \gg 1$ ). A narrow air gap of length $\ell_g$ is cut in the core. A coil of $N$ turns carrying current $I$ is wound uniformly around the toroid. Assume the magnetic flux is confined to the core/gap, and neglect fringing so that $\vec{B}$ is uniform and the same in the core and in the gap.

(a) Find $H$ and $B$ in the core and in the air gap.

(b) Find the magnetic energy stored in the core and in the gap.

(c) Find the inductance $L$ of the gapped toroid. For the convenience of derivation, you may define an equivalent length as

$$l_e = l_g + \frac{2\pi\rho - l_g}{\mu_r}.$$




**Problem 6: 15 points**

A uniform time-harmonic plane wave with electric field $E_0 e^{-j\beta_0 z}\hat{a}_x$ is normally incident from Region 1 onto a lossless dielectric slab (Region 2), as described below:

- Region 1: $z < 0$, free space with intrinsic impedance $\eta_0$.
- Region 2 (slab): $0 \leq z \leq d$ , lossless dielectric with intrinsic impedance $\eta = 2\eta_0$. The slab thickness is $d = \lambda_2/8$, where $\lambda_2$ is the wavelength of the wave in Region 2.
- Region 3: $z > d$, free space again.

(a) Find the overall reflection coefficient seen in Region 1 at $z = 0$.

(b) Find reflected and transmitted power fraction $R = P_r/P_i$ in Region 1 and the transmitted power fraction $T = P_t/P_i$ in Region 3.

(c) Determine the standing-wave ratio (SWR) in Region 1.

**Problem 7: 15 points**

An antenna located at the origin has a far-zone electric field in free space as

$$\vec{E} = \frac{\cos 2\theta}{r} e^{-j\beta r} \hat{a}_\theta \ \mathrm{V/m}$$

with time dependence $e^{j\omega t}$ and intrinsic impedance $\eta_0 = 120\pi\ \Omega$.

(a) Obtain the corresponding magnetic field $\vec{H}$.

(b) Determine time-average power density in units of $\mathrm{W/m^2}$.

(c) Determine the total power radiated by the antenna.

(d) Determine the radiation resistance of the antenna if the peak of the feed current for the antenna is 1 A.

(e) Determine the directivity of the antenna.

The following integral may be useful in your calculation:

$$\int_0^\pi \cos^2(2\theta) \sin\theta d\theta = \frac{14}{15}.$$