國立成功大學
115學年度碩士班招生考試試題

編　號：126

系　所：電機工程學系

科　目：線性代數

日　期：0203

節　次：第 3 節

注　意：1.不可使用計算機
　　　　2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (40 pts, 4 pts each) Mark each statement True or False (2 pts for correct answer). Justify each answer (2 pts).
   a. Suppose that A is a rank 10 matrix with size 13×20. Let $A = UDV^T$ be a reduced SVD for A. Then D has size 13×13.
   b. Suppose that A is a rank 10 matrix with size 13×20. Let $A = UDV^T$ be a reduced SVD for A. Then U has size 13×10.
   c. Suppose that A is a rank 10 matrix with size 13×20. Let $A = UDV^T$ be a reduced SVD for A. Then V has size 10×20.
   d. The matrix A is symmetric and has the characteristic polynomial $p(\lambda)=\lambda^3(\lambda-1)^2(\lambda+3)$. Then Dim Nul $A = 2$.
   e. Let Q be an $n \times n$ orthogonal matrix. Then $Q^2(Q^T)^3Q^{-1}(Q^T)^{-1}(Q^{-1})^T = Q$.
   f. If A is an $n \times n$ orthonormal matrix and B is an $n \times n$ matrix, then the equation $AX = B$ has a unique solution.
   g. Let W be a subspace of $\Re^n$ and let $x \in W$. Then $\text{Proj}_W(x) = 0$.
   h. Let A and B be $n \times n$ matrices. Then $\det(A+B) = \det A + \det B$.
   i. Suppose that matrices A and B satisfy $AB = \mathbf{0}$. Then either $A = \mathbf{0}$ or $B = \mathbf{0}$.
   j. Let V be a subspace of $\Re^m$ and let $y \in V$. Then $\text{Proj}_V(y - \text{Proj}_V(y)) = 0$.

2. (40 pts, 10 pts each) Find the following factorization, diagonalization, decomposition.
   a. $A = \begin{bmatrix} 1 & -2 & -4 & -3 \\ 2 & -7 & -7 & -6 \\ -1 & 2 & 6 & 4 \\ -4 & -1 & 9 & 8 \end{bmatrix}$, Find an LU factorization of A.
   b. $A = \begin{bmatrix} 7 & 4 & 16 \\ 2 & 5 & 8 \\ -2 & -2 & -5 \end{bmatrix}$. Diagonalize A into the form PDP^{-1}.
   c. $A = \begin{bmatrix} -2 & 3 \\ 5 & 7 \\ 2 & -2 \\ 4 & 6 \end{bmatrix}$. Find a QR factorization.
   d. $A = \frac{1}{15}\begin{bmatrix} 19 & 14 & 10 \\ 8 & -2 & 20 \end{bmatrix}$. Find the SVD of A ($A = U\Sigma V^T$).

3. (20 pts) Let $t_0, \ldots, t_n$ be distinct real numbers. For p and q in P_n, define $\langle p, q\rangle = p(t_0)q(t_0) + p(t_1)q(t_1) + \ldots + p(t_n)q(t_n)$, and note that $\langle p, p\rangle = [p(t_0)]^2 + [p(t_1)]^2 + \ldots + [p(t_n)]^2 \geq 0$, and $\langle \mathbf{0}, \mathbf{0}\rangle = 0$. Let P_3 have the inner product given by evaluation at -3, -1, 1, and 3. Let $p_0(t) = 1$, $p_1(t) = t$, and $p_2(t) = t^2$.
   a. Compute the orthogonal projection of p_2 onto the subspace spanned by p_0 and p_1. (10 pts)
   b. Find a polynomial q that is orthogonal to p_0 and p_1 such that $\{p_0, p_1, q\}$ is an orthogonal basis for span $\{p_0, p_1, p_2\}$. Scale the polynomial q so that its vector of values at (-3, -1, 1, 3) is (1, -1, -1, 1). (10 pts)