

1. (a) Find the general solution of $y''' - y'' - 8y' + 12y = 7e^{2x}$
 (b) Find a function $f(t)$ satisfying $f(t) = e^{-t} + 2 \int_0^t e^{-3\tau} f(t-\tau) d\tau$

2. Find the eigenvalues and eigenvectors of the matrix

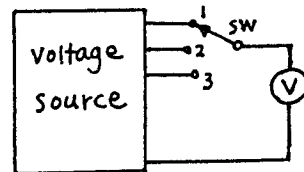
$$A = \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 2 & 0 \\ \sqrt{2} & 0 & 0 \end{bmatrix}$$

3. Given $\oint_C \frac{e^{3z}}{(z-\delta_0)^4} dz = 9\pi i e^{-\pi}$
 where C is the circle $|z| > \delta_0$. Find δ_0 .

4. At some instant, the switch sw may stay at any one of the three positions. The probability $P[X]$ of the switch position X is

$$P[X=1] = \frac{1}{4}, P[X=2] = \frac{1}{4}, P[X=3] = \frac{1}{2}$$

The distribution of the voltage V measured at position X is



- position 1 : uniformly distributed over 0 and 2 volts
- position 2 : uniformly distributed over 1 and 3 volts
- position 3 : uniformly distributed over 0 and 4 volts

- (a) Find the probability $P[V \leq 2]$
 (b) Given $V > 2$ volts, find the probability that the switch is at position 3.

5. (a) Find the Laplace transform of the function $f(t) = \begin{cases} 1 & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \\ \cos t & \text{if } t > 2\pi \end{cases}$
 (b) Find $\mathcal{L}^{-1} \left[\frac{1}{s(s-a)^2} \right]$ by convolution method.

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}[\cos \omega t] = \frac{s}{s^2 + \omega^2}$$

$$\mathcal{L}[e^{at} t^n] = \frac{n!}{(s-a)^{n+1}}$$