國立成功大學76學年度碩研所 考試(工程數學 試題) 共一頁 第一頁

研究所碩士班入學

1. Solve the equation 10%

$$y'' - y = x e^x \sin x$$

2. Given 15%

$$f(x) = \begin{cases} 0 & \text{when } -\infty < x < -\pi \\ -1 & \text{when } -\pi < x < 0 \\ 1 & \text{when } 0 < x < \pi \\ 0 & \text{when } \pi < x < \infty \end{cases}$$

(a) Determine the Fourier integral for the function

(b) To what number does the integral found in (a) converge at $x = -\pi$

3. Find the solution of Bessel's equation 20%

$$x^2 y''(x) + x y'(x) + x^2 y(x) = 0$$

by the Laplace Transform method.

4. Evaluate 15%

$$I = \iint_S (x^3 dy dz + x^2 y dz dx + x^2 z dx dy)$$

where S is an open surface consisting of right circular cylinder $x^2+y^2=1$, $0 \le z \le 1$ and the circular disc $x^2+y^2=1$, $z=0$

5. If $\lambda_1, \lambda_2 \cdots \lambda_n$ are the eigenvalues of a square matrix A, and if f is any polynomial. Prove that the determinant of $f(A)$ is 10%

$$|f(A)| = f(\lambda_1) f(\lambda_2) \cdots f(\lambda_n)$$

6. Show that 15%

$$\text{P.V.} \int_{-\infty}^{\infty} \frac{e^{itx}}{x} dx = \begin{cases} \pi i & (t>0) \\ 0 & (t=0) \\ -\pi i & (t<0) \end{cases}$$

(Note: P.V. ≡ Principal value)

7. Find the solution $u(x,t)$ for the following initial-boundary value problem 15%

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \qquad t>0 \qquad 0<x<1$$

I.C. $u(x,0) = f(x)$ $\quad 0 \le x \le 1$

B.C. $u(0,t) = 0$, $u(1,t) = u_0$ $\quad t \ge 0$

where the function $f(x)$ satisfies the following compatibility conditions $f(0)=0$, $f(1)=u_0$