國立成功大學77學年度光機研究所碩士班考試(工程數学 試題) 共 1 頁 第 1 頁

1. Find the solutions of

(a) $u_{xy} = x^2 y$ when $u_y(0, y) = y^2$ and $u(x, 1) = \cos x$, where $u = u(x, y)$. (10%)

(b) $x^2 y'' + 7xy' + 9y = 0$ when $y(1) = 1$ and $y'(0) = 0$, where $y = y(x)$. (10%)

2. (a) Find the Fourier series for $f(x) = |\sin x|$, $-\pi < x < \pi$. (8%)

(b) Calculate $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2-1}$ (6%)

3. (a) Find the Laplace transform of $(e^{at} - e^{bt})/t$ for $t \geq 0$, indicating the region of convergence of the integral involved, where a, b are constants. (8%)

(b) Find the inverse Laplace transform of $\tan^{-1}\frac{1}{s}$ for $s \geq 0$. (8%)

4. If the matrix A is

$$\begin{bmatrix} 25 & 40 \\ -12 & -19 \end{bmatrix}$$

, show that $A^n \begin{pmatrix} 3 \\ -2 \end{pmatrix} = 5^n \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

, where n is a positive integer. (10%)

5. Evaluate the integral $\int \vec{n} \cdot \operatorname{curl} \vec{F}\, dS$ for the vector function

$$\vec{F} = (2y^2 + 3z^2 - x^2)\vec{i} + (2z^2 + 3x^2 - y^2)\vec{j} + (2x^2 + 3y^2 - z^2)\vec{k}$$ (14%)

over the part of the surface $x^2 + y^2 - 2ax + az = 0$ which lies above the plane $z = 0$, where $\vec{n}$ is the unit outer normal of the surface and $\operatorname{curl}\vec{F}$ is the curl of $\vec{F}$.

6. Calculate $\int_{C_1} \tan(\pi z)\, dz$ and $\int_{C_2} \frac{\sin z}{(z-\pi)^2}\, dz$, where $z = x + iy$ is a complex variable. C_1: $4(x-\frac{1}{2})^2 + 9y^2 = 1$ and C_2 (as shown in Fig. 1) are in the counterclockwise direction. (16%)

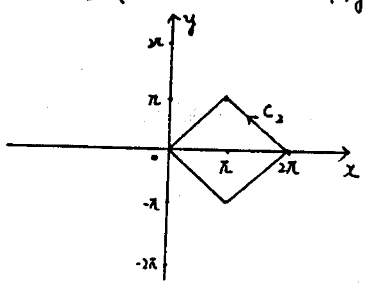

Fig. 1

7. Assume that X and Y are independent random variables with the following probability density functions

$$X:\ f(x) = \begin{cases} \frac{x}{2}\ ; & 0 \leq x \leq 2 \\ 0\ ; & \text{elsewhere} \end{cases} \qquad Y:\ g(y) = \begin{cases} \frac{y^3}{64}\ ; & 0 \leq y \leq 4 \\ 0\ ; & \text{elsewhere} \end{cases}$$ (10%)

Calculate the probability density function of $W = XY$.