78共13張

國立成功大學 78 學年度電机研究所碩士班考試( 工程數學 試題) 共 1 頁 第 1 頁

1 Solve the equation

$$x^2y''-5xy'+8y=2x\ln(x)+x^3 \quad (x>0)$$ (15%)

2. Find a function $f(x)$ satisfying $f(t)=2t^2+\int_0^t \sin(4x) f(t-x)\,dx$. (15%)

3. (a) Find the Fourier series of a periodic function whose definition in one period is

$$f(x)=\begin{cases} 0 & -\pi \le x \le 0 \\ \sin x & 0 \le x \le \pi \end{cases}$$ (10%)

(b) Calculate : (i) $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\frac{1}{7\times9}+\cdots$ (5%)

(ii) $\frac{1}{1\times3}-\frac{1}{3\times5}+\frac{1}{5\times7}-\frac{1}{7\times9}+\cdots$ (5%).

4. (a) If $|\vec{A}|=2$, $|\vec{B}|=4$, and $|\vec{C}|=10$, can $\vec{A}+\vec{B}+\vec{C}=\vec{0}$ (null vector)? Explain your answer briefly. (5%)

(b). Find the derivative of $f=xyz$ at the point $(1,3,2)$ in the direction of the vector $2\hat{i}-\hat{k}$. What is the maximum possible directional derivative of $f$ at that point, and what is its direction? What is the equation of the tangent plane to the surface $f=$ constant at the point $(1,3,2)$? (6%)

(c) Evaluate $\int_S \vec{v}\cdot\vec{n}\,da$, where $S$ is the surface of the cylinder $x^2+y^2\le4$, $0\le z\le3$, and $\vec{v}=x\hat{i}+y\hat{j}+z\hat{k}$. (6%)

5. (a) Determine the characteristic numbers $(\lambda_1,\lambda_2)$ and corresponding Hermitian unit eigenvectors $(\hat{e}_1,\hat{e}_2)$ of the problem

$$9x_1+(2+2i)x_2=\lambda x_1$$
$$(2-2i)x_1+2x_2=\lambda x_2$$

where $i^2=-1$, and verify that $\hat{e}_1$ and $\hat{e}_2$ are orthogonal in the Hermitian senes. (6%)

(b) If $\overleftrightarrow{H}$ is the coefficient matrix of the system of part (a) and $\overleftrightarrow{U}$ is the orthonormal modal matrix made up of $\hat{e}_1$ and $\hat{e}_2$, verify that:

$$\overleftrightarrow{U}^*\overleftrightarrow{H}\,\overleftrightarrow{U}=[\lambda_i\delta_{ij}].$$ (6%)

(c) If $\vec{V}=(1+i,\ 1)$, determine $\alpha_1$ and $\alpha_2$ such that

$$\vec{V}=\alpha_1\hat{e}_1+\alpha_2\hat{e}_2$$

where $\hat{e}_1$ and $\hat{e}_2$ are the vectors determined in part (a). (6%)

6. Evaluate $\int_{-\infty}^{\infty}\frac{\sin^3 x}{x^3}\,dx$ (15%)

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