

1. The block diagram of a unity feedback control system is shown in Fig. 1.

- (a) Determine the gain margin in dB when the phase-crossover frequency  $\omega_x < 10$ .
- (b) Determine the stability of this system.
- (c) Draw the Bode plot of the open-loop transfer function.

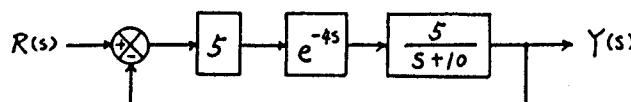


Fig. 1

2. Given a system  $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -16 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$  and  $\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ,  $y = [1 \ 4] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

- (a) Find a Lyapunov function  $V(x)$  for the system with  $-\dot{V}(x) = x_1^2 + 4x_1x_2 + 10x_2^2$ . Determine the stability using the Lyapunov's method.

(b) Evaluate  $J = \int_0^\infty [x_1^2(t) + 4x_2^2(t)] dt$  and  $J = \int_0^\infty y^2(t) dt$

3. Obtain the transfer function  $Y(s)/R(s)$  of the system shown in Fig. 2.

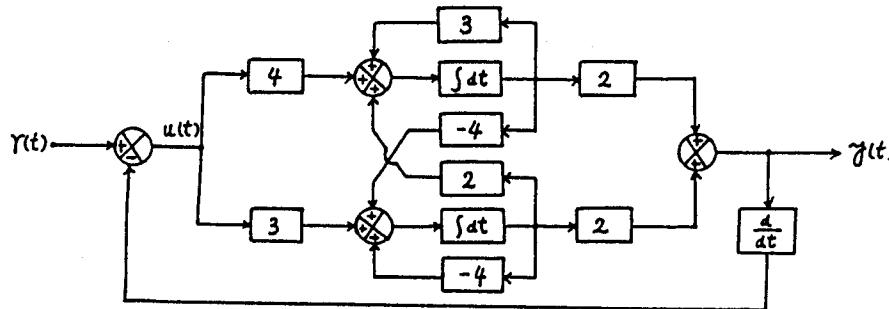


Fig. 2

4. The block diagram of a second-order system is shown in Fig. 3. Suppose that the time-domain response equation of this system to the input  $r(t) = 2u(t)$  is given as  $c(t) = 5[1 - xe^{-\gamma t} \sin(500t + 1)]$ .

- (a) Calculate the values of  $x$  and  $y$ .
- (b) Determine the values of system parameters  $A$ ,  $B$ , and  $D$ .
- (c) Determine the system transfer function using the parameter values.

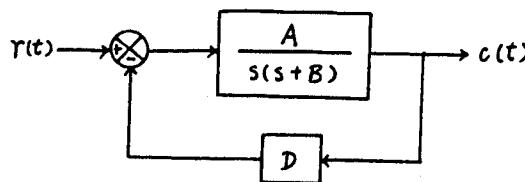


Fig. 3