國立成功大學 79 學年度 電研所 考試(工程數學 試題) 共 1 頁 第 1 頁

1. Solve the equation

$$y'' + y = \csc(x) \qquad (16\%)$$

2. Find the Fourier series of the function $f(x)$ as

$$f(x) = 1 - |x|, \quad -3 \le x \le 3 \qquad (16\%)$$

3. A R-L-C circuit and the voltage source $E(t)$ are shown in Fig. 1 and Fig. 2, respectively. If $I(0)=0$, solve the $I(t)$ by using Laplace transform. (18%)

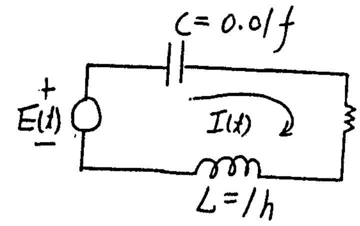

Fig. 1.

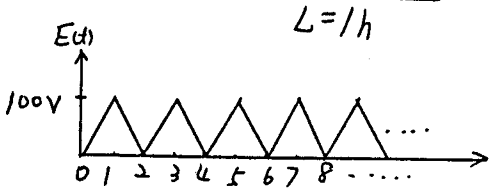

Fig. 2

4. Two lines are defined as follow:

$$L_1: \begin{cases} x = 1+2t \\ y = 1+3t \\ z = 2-t \end{cases} \qquad L_2: \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}$$

(a) Find the shortest distance between L_1 and L_2.

(b) Find the equation of the plane which passes through $(1, 0, 1)$ and parallel to lines L_1 and L_2. (10%)

5. Solve the following boundary-valued problem:

$$\nabla^2 u = u_{xx} + u_{yy} = 0 \qquad 0 < x < \pi, \quad 0 < y < \pi$$

$$u_x(0, y) = 0, \quad 0 < y < \pi; \qquad u(\pi, y) = C_1, \quad 0 < y \le \pi;$$

$$u_y(x, 0) = 0, \quad 0 < x < \pi; \qquad u(x, \pi) = C_2, \quad 0 < x < \pi;$$

where $u_x = \dfrac{\partial u(x,y)}{\partial x}$, $u_y = \dfrac{\partial u(x,y)}{\partial y}$; C_1 and C_2 are real constants. (14%)

6. Determine the mapping of $x^2+y^2 \le 1$ under the transformation

$$W = \ln\left(\frac{1+z}{1-z}\right); \quad z = x + iy \qquad (12\%)$$

7. Two matrices U and H are related by $U = e^{iaH}$, where $i = \sqrt{-1}$; a is a real constant. (The exponential matrix function may be interpreted by a Maclaurin expansion.)

(a) If H is Hermitian, show that U is unitary.

(b) If U is unitary, show that H is Hermitian. (14%)

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