

1. (a) Draw the Nyquist diagram for the system shown in Fig. 1.  
 (b) Estimate the range of  $K$  based on the Nyquist stability analysis for which the system is stable, and verify with a root locus plot.

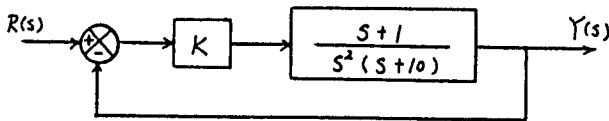


Fig. 1

2. Design the system shown in Fig. 2 (i.e., find  $K_p, J, D$ ) that satisfies the following specifications:  
 (a) The steady-state error  $e_{ss} < 0.05$  rad for step input  $\theta_m = 1.0$  rad (with  $T_d = 0$ ).  
 (b)  $e_{ss} < 0.05$  rad for ramp input  $\dot{\theta}_m = 75$  rad/sec (with  $T_d = 0$ ).  
 (c)  $e_{ss} < 0.05$  rad for step disturbance  $T_d = 50$  N·m (with  $\dot{\theta}_m = 0$ ).  
 (d) Setting time  $T_s = 0.1$  sec for 5% transient residue. (Hint:  $T_s = 3/\zeta\omega_n$ )  
 (e) Phase margin  $PM = 45^\circ$ .

The margins of stability are shown in the Table 1.

(Hint:  $\frac{\theta_{out}(s)}{\theta_m(s)} = \frac{K_p}{Js^2 + Ds + K_p} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$  when  $T_d(s) = 0$ )

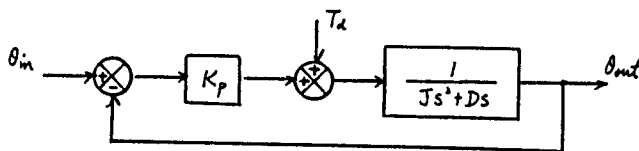


Fig. 2

$\zeta$	PM	GM
0	$0^\circ$	0
0.125	$20^\circ$	6.3
0.25	$30^\circ$	25
0.5	* $45^\circ$	100
0.707	$60^\circ$	200
1.0	$90^\circ$	400

Table 1

3. Let  $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 1 & 2 & 3 \\ 2 & 1 & 0 & 0 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 2 \\ 0 & 2 \end{pmatrix}$

Find two different real constant  $2 \times 4$  matrices  $K$  such that the matrix  $(A+BK)$  has eigenvalues  $-4 \pm 3j$  and  $-5 \pm 4j$ .

4. Consider the system defined by the equations

$$x_1(k+1) = 2x_1(k) + 0.5x_2(k) - 5$$

$$x_2(k+1) = 0.8x_2(k) + 2$$

Determine the stability of the equilibrium state.