國立成功大學八十學年度電機工程研究所考試(工程數學 試題) 共 1 頁 第 1 頁

1. Solve the differential equation

$$y'' + 2y' + y = -3e^{-x} + 8xe^{-x} + 1 \qquad (16\%)$$

2. (a) Find the Fourier series of a periodic function $f(x)$ defined as

$$f(x) = \begin{cases} K\sin(Tx), & 0 < x < \frac{\pi}{T} \\ 0, & -\frac{\pi}{T} < x < 0 \end{cases}$$

and $f(x + \frac{2n\pi}{T}) = f(x)$, $n = 1, 2, 3, \ldots$ (12%)

(b) Evaluate the infinite series

$$\frac{1}{1\times3} - \frac{1}{3\times5} + \frac{1}{5\times7} - \frac{1}{7\times9} + \cdots \qquad (6\%)$$

3. Solve the differential equation with variable coefficients

$$ty'' - ty' - y = 0, \quad y(0) = 0, \; y'(0) = 3 \qquad (16\%)$$

4. A tool manufacturer ships equal quantities of hammers, pliers, and wrenches. If the probabilities of defective tools to be 0.005 for hammers, 0.003 for pliers, 0.008 for wrenches. If a tool is found to be defective, calculate the probability that is a hammer. (10%)

5. Define $f(z) = \frac{1}{(z-1)(z-3)}$, where z is a complex variable. If $|z| > 1$, find all the Laurent series of $f(z)$. (14%)

6. For $\vec{F} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4z\vec{k}$, find the volume integral of $\nabla \cdot \vec{F}$ over the volume enclosed by $z = 4 - x^2$, $y = 0$, $y = 3$, $x = 0$ and $z = 0$. (13%)

7. Find the orthonormal eigenvectors of the matrix

$$B = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{pmatrix} \qquad (13\%)$$

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