This entrance examination includes three parts: Part I: Digital Control (25%); Part II: Linear Systems (25%); Part III: Classical Control (50%).

Part I: Digital Control

1. Find the final value of $C(z) = z^2/[(z-1)(z^2-0.4z-0.96)]$. (5%)

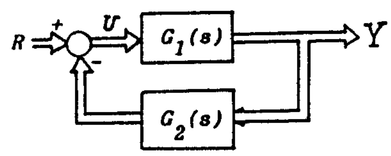
2. For a standard 2nd-order system, the poles in z-plane are $z_{1,2} = re^{\pm j\theta}$, the associated poles in s-plane are $s_{1,2} = -\varsigma\omega_n \pm j\omega_n\sqrt{1-\varsigma^2}$, determine ς and ω_n, if r, θ, and the sampling period T are given. (8%)

3. The characteristic equation of a system is $1+KG(z) = 0$, where $G(z) = Z[G_h(s)G_o(s)]$, $G_h(s)$ is the transfer function of Zero-Order-Hold, $G_o(s) = 1/[s(s-1)]$, and the sampling period $T = 1Sec.$. Use *Jury's stability test* to determine the range of K. (12%)

Part II: Linear Systems

4. *Lemma* : Let $G_1(s)$ and $G_2(s)$ be, respectively, $q \times p$ and $p \times q$ rational function matrices. Then we have $det(I_p + G_2(s)G_1(s)) = det(I_q + G_1(s)G_2(s))$, where I_p and I_q are $p \times p$ and $q \times q$ identity matrices, respectively.

Theorem : Consider the following feedback systems, if $det(I_q + G_1(s)G_2(s)) \neq 0$, then the transfer function matrix of the feedback system is given by



$G_{CL}(s) = G_1(s)(I_p + G_2(s)G_1(s))^{-1}$
$= (I_q + G_1(s)G_2(s))^{-1}G_1(s)$.

Prove this *Theorem*.

(*Hint:* you can use and do not have to prove the *Lemma*.) (10%)

5. *Definition*: Let $\Psi(t)$ be any fundamental matrix of $\dot{X} = A(t)X(t)$, then $\Phi(t,t_o) \equiv \Psi(t)\Psi(t_o)^{-1}$, for all t, t_o in $(-\infty, +\infty)$, is the state transition matrix of $\dot{X} = A(t)X(t)$.

(a) From $\Phi(t,\tau)$, show how to compute $A(t)$. (8%)

(b) Show that $\Phi(t,t_o)$ is uniquely determined by $A(t)$ and is independent of the $\Psi(t)$ chosen. (7%)

Part III: Classical Control

6. Given the unit-step feedback system shown in Fig. 1 where

$$G(s) = \frac{1}{s(1+0.5s)(1+s)},$$

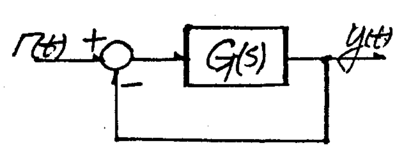


Fig. 1

(i) find the gain margin G_m, (10%)

(ii) determine the stability of the closed-loop system based on the Nyquist stability criterion. (Note that some necessary explanations are required, and the entire Nyquist plot in not necessary.) (10%)

7. Given the system shown in Fig.2, plot the output responses at steady state for $r(t)$ being a unit-step function $r(t) = u(t)$ and a ramp function $r(t) = tu(t)$, respectively. (Hint: Determine the steady-state error first.) (10%)

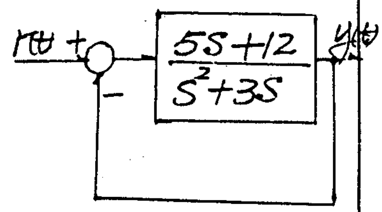


Fig.2

8. The block diagram of a control system is shown in Fig.3. Draw the root locus plot of the system with K as varying parameter ($0 \le K \le \infty$) and particularly show (i) the starting and ending points, (ii) the real-root branches, (iii) the center of gravity, (iv) the asymptotics, (v) the breakaway points and the breakaway gain (K_b), (vi) the marginal gain (K_m) and the associated roots on the imaginary axis.

(20%)

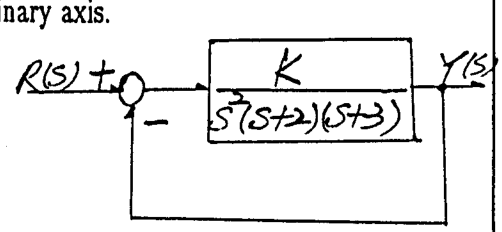


Fig. 3

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