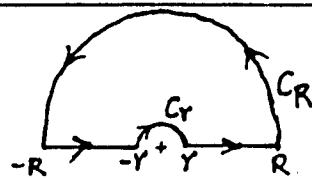


1. (a) Evaluate $\int_0^{\infty} \frac{x^2}{x^2+1} dx$ (10%)

(b) Evaluate $\int_0^{\infty} \frac{\ln(x)}{x^2+1} dx$ by using (10%)

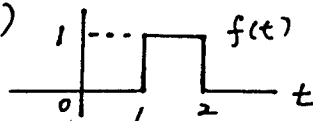


the closed contour shown on the right.

Note that \ln is a natural logarithmic function (i.e., $\ln(e^{i\theta}) = i\theta$) and $0 \leq \theta < 2\pi$. Also, note that $\int_0^{\infty} \frac{1}{x^2+1} dx = \pi/2$.

2. Using Laplace Transform to solve (15%)

$$-ty' + 2y = t f(t)$$



$f(t)$ is defined as $f(t) = 1$ if $1 \leq t \leq 2$ and $f(t) = 0$ elsewhere.

3. Find the series solution of $4x^2 y'' + 2xy' - xy = 0$ (10%)

4. Solve the boundary value problem (15%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u \quad (x > 0, t > 0); \quad \frac{\partial u}{\partial x}(0, t) = -e^{-t}; \quad u(x, 0) = 0$$

5. A is a Hermitian matrix, while B is a skew-Hermitian matrix. Let $C = AB$. Is C Hermitian, or skew-Hermitian, or none of both? (10%)

6. Find the tangent plane and normal line to (10%)
the surface of $z^2 = x^2 - y^2$ at the point $(1, 1, 0)$

7. $f(x) = e^{2|x|}$ if $-1 < x < 1$ and $f(x+1) = f(x-1)$. The Fourier series of $f(x)$ is $f(x) = \frac{1}{2}(e^2 - 1) + \sum_{n=1}^{\infty} \frac{4}{4+n^2\pi^2} [e^2 \cos(n\pi) - 1] \cos(n\pi x)$. Using this known information, answer the following questions.

(a) Find the Fourier series of $g(x)$, which is defined as $g(x+1) = g(x-1)$, $g(x) = e^{2x}$ if $0 < x < 1$, and $g(x) = -e^{-2x}$ if $-1 < x < 0$ (5%).

(b) Find the complex Fourier series of $g(x)$. (5%)

(c) Find the Fourier Transform of $h(x)$, which is defined as $h(x) = g(x)$ if $-1 \leq x \leq 1$, but $h(x) = 0$ if $|x| > 1$. (5%)

(d) Find the Fourier Transform of $\frac{dh(x)}{dx}$ (5%).