研究所入學考離散數學試題

1\.
a. Find the simplified generating function of the following sequence:

$$C(n,0),\ C(n,2),\ \cdots,\ C(n,q)$$

b. Verify that

$$C(n,0)+C(n,2)+\cdots+C(n,q)=\begin{cases}2^{n-1} & \text{for } n>0\\ 1 & \text{for } n=0\end{cases}$$

for $q=\begin{cases} n & \text{if } n \text{ is even}\\ n-1 & \text{if } n \text{ is odd.}\end{cases}$

2\. Define the circuit rank of a disconnected graph to be the sum of the circuit ranks of all its connected components. Derive a formula for the circuit rank of $G=(V,E)$ involving $|E|$, $|V|$, and $C(G)$ which is the number of components of G.

3\. Write an expression for a_r, where a_r is the coefficient of X^r in the following generation function $A(X)$:

$$\frac{3}{(1-X)^2}-\frac{7}{(1-2X)^3}+\frac{8}{3+2X}$$

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4. Let a_r be the number of ways the sum r can be obtained by tossing 50 distinguishable dice. Write a generating function for the sequence $\{a_r\}_{r\geq 0}^{\infty}$. Then find the number of ways to obtain the sum of 100, that is, find a_{100}.

5. Use the fact that every polynomial equation having real-number coefficients and odd degree has a real root in order to show that the function $f: R \to R$, defined by $f(x) = x^5 - 2x^2 + x$, is an onto function. Is f one-to one?

6.
a: Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{w, x, y, z\}$, and $D \subseteq A \times B$. If π_A and π_B are both to be onto functions, What is the minimal value that is possible for $|D|$?

b: For A, B as in part (a), how many such different minimal sets D result in both π_A and π_B being onto?

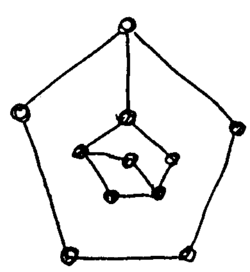
c: Generalize the results of parts (a) and (b).

7. Show that we cannot construct a finite state machine that recognizes precisely those sequences in the language $A = \{0^i 1^j \mid i, j \in \mathbb{Z}^+, i < j\}$. (The alphabet for A is $\Sigma = \{0, 1\}$)

8. Find the coefficient of x^{83} in
$f(x) = (x^5 + x^8 + x^{11} + x^{14} + x^{17})^{10}$

9. In how many ways can the letters PENTAGONAL be arranged with exactly two consecutive vowels?

10. Prove that the following graph does not have any Hamilton circuit.



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