83 共 20 張

國立成功大學 83 學年度電机碩士考試（工程數學試題）

1. Please give an example to describe briefly the relationship between the single-valued function and the analytic function. (10%)

2. Determine the integrals. (20%)

(i) $\int_0^\infty \cos(x^3)\,dx$ (Assume $\int_0^\infty e^{-x^n}\,dx = An$)

(ii) $\int_0^\infty \frac{\ln(x)}{(x^2+1)^2}\,dx$

3. Consider the set of vectors $\{f_1, f_2, f_3\}$ in R^4, where
$f_1=(1, 0, 1, 0)$; $f_2=(2, 1, 2, 1)$; $f_3=(0, 2, -2, 2)$
(i) Find an orthonormal set of vectors $\{g_1, g_2, g_3\}$ such that $\{f_1, f_2, f_3\}$ and $\{g_1, g_2, g_3\}$ generate the same subspace U in R^4. (8%)
(ii) Find a vector X in R^4 such that X is perpendicular to U and $\|X\|=1$. (4%)

4. Let $T: P_2 \to R^4$ be the function defined by the formula

$$T(P(x))=\begin{bmatrix} P(0) \\ P(1) \\ P(2) \end{bmatrix}$$, where $P_2= C_0 + C_1 x + C_2 x^2$ and P(x) belongs to P_2

(i) Is T a linear transformation? (4%)
(ii) Is T one-to-one? (4%)
Prove your answers.

5. Expand $f(x)=\begin{cases} +1, & 0 < x < 1 \\ -1, & -1< x < 0 \end{cases}$ in a Fourier-Legendre series, $f(x)=\sum_{n=0}^{\infty} C_n P_n(x)$

(Hint: use orthogonal relation $\int_{-1}^{1} P_m(x)P_n(x)dx=\left[\frac{2}{2m+1}\right]\delta_{mn}$

and $P_m(x)=\frac{1}{(2^m)m!}\frac{d^m}{dx^m}(x^2-1)^m$). (10%)

6. By use of the convolution theorem, find
$L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}=?$ (L^{-1} is inverse Laplace transform). (10%)

7. Evaluate $\oint_C x^2y\,dx - xy^2\,dy$,
C is the boundary of the region $x^2+y^2 \le 4$, $x\ge0$, $y\ge0$. (10%)

8. Solve the Partial differential equation by using a Fourier Transform. (20%)

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + t\,u = 0 \quad (x>0,\ t>0)$$

$u(0, t)=0$ $(t>0)$
$u(x, 0)=e^{-x}$ $(x>0)$
assume that Fourier sine transform of e^{-ax} is equal to $\omega/(a^2+\omega^2)$