* Useful constants :

$\varepsilon_0 = 10^{-9}/(36\pi)$ (F/m); $\mu_0 = 4\pi \times 10^{-7}$ (H/m); $\sqrt{\mu_0/\varepsilon_0} = 120\pi$ (Ω)

1. **Maxwell's Equations : (20%)**

(a) Use differential forms to explain how did Maxwell modify the Guass's law, Ampere's law, and Faraday's law to the so-called Maxwell's equations.
(b) Indicate which equation and how did Maxwell's used to modify the above laws.
(c) Which term did he add? Explain the physical meaning of that term.
(d) Derive the nonhomogeneous wave equations for the vector E and H field from Maxwell's equations.
(e) Derive the homogeneous (source-free) Helmholtz's equation.

Note : $\nabla \times \nabla \times \bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$

2. The far-zone E field of a Hertzian electric dipole of current I_H and length L along the z-axis is

$$E_\theta = j\eta_0 \beta I_H L\left(\frac{e^{-j\beta r}}{4\pi r}\right)\sin\theta$$

and that of an element magnetic dipole of loop current I_M and radius b lying in the xy-plane is

$$E_\phi = \omega\mu_0\beta m\left(\frac{e^{-j\beta r}}{4\pi r}\right)\sin\theta \qquad m = I_M(\pi b^2)$$

(20%)

(a) Derive the far-zone H fields of the Hertzian dipole and the element magnetic dipole.
(b) Prove that a composite antenna consists of a Hertzian dipole and an element magnetic dipole can produce a circularly polarized wave. Also Indicate the necessary conditions.
(c) Derive the radiation resistance of a Hertzian dipole and a element magnetic dipole, respectively.

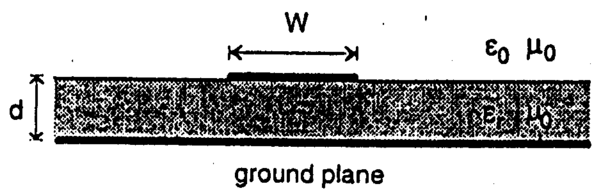
Note : $\int_0^\pi \sin^3\theta d\theta = 4/3$

3. A microstrip transmission line is shown in the following figure. (20%)

(a) Explain why the microstrip line can not support a pure TEM guided wave.

(b) It is usually assumed that quasi-TEM wave in a microstrip line which means that the fields are essentially the same as those of the static case. Draw a figure to illustrate an approximated E and H field distribution over the cross section of the microstrip line.

(c) Write the most simple formula of the microstrip line characteristic impedance Z_0 (assuming no conductor and dielectric loss). What is the basic assumption (related to W and d) to achieve this simple formula?



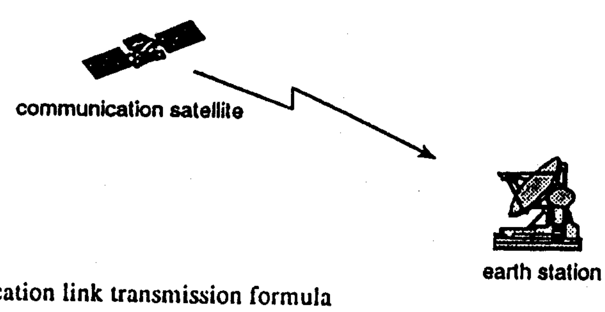
Microstrip Transmission Line

4. A communication satellite is in a synchronous orbit 36,000 km above the earth. The satellite transmitter power is 10 W and the antenna has a gain 30 dB and radiation efficiency 0.6 at 4 GHz. The earth station receiver with an antenna gain 40 dB and radiation efficiency 0.5 receives the satellite microwave signal through the horn feed and the waveguide. (20%)

(a) Determine the time-average radiation power density of the satellite antenna at the location of the earth station.

(b) Determine the received power of the earth station antenna (assuming a matched condition and no polarization mismatch).

(b) Let the **a** x **b** rectangular waveguide (**a** > b) is to operate in dominant mode and the cutoff frequency is 25% lower than the operating frequency 4 GHz. Determine the dimension **a**.



Communication link transmission formula

$$\frac{P_L}{P_{rad}} = \frac{G_t \eta_t G_r \eta_r \lambda^2}{(4\pi r)^2}$$

P_{rad} = radiated power from the antenna , P_L = received power to the matched load

5. **Waveguide Problem**

An automotive tunnel is rectangular in cross section(10 m wide and 5 m high) and is modeled as a waveguide (assuming the metallic tunnel wall). (20%)

(a) Will radio signals in the AM broadcast band (550 to 1600 KHz) travel in this tunnel? If yes, what is the necessary condition for the polarization of the incident radio wave?
*Note : you have to make some calculation and explain the reason in detail.

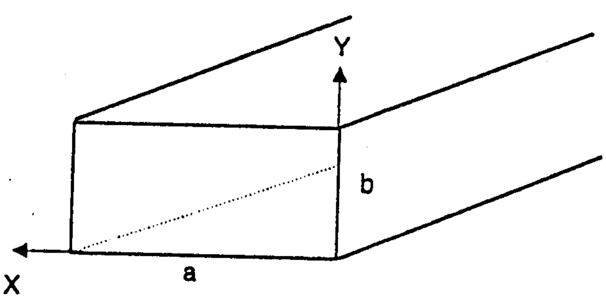
(b) Will radio signals in the FM broadcast band (88 to 108 MHz) travel in this tunnel? If yes, what is the necessary condition for the polarization of the incident radio wave?

(c) List all TE and TM waveguide modes which will propagate in the tunnel for a 40-MHz radio wave signal.

(d) The attenuation constant of the waveguide due to the metallic wall losses for the dominant mode (in this case) is

$$\alpha_c = \frac{1}{\eta b}\sqrt{\frac{\pi f \mu_c}{\sigma_c[1-(f_c/f)^2]}}\left[1+\frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right] \qquad (Np/m)$$

Let $\mu_c = \mu_0$, $\sigma_c = 5\times10^8$, and the tunnel length = 100m. Determine the total attenuation (dB) of the 40-MHz radio wave signal (in dominant mode) propagating through the tunnel.



An automotive tunnel modeled as a waveguide.

Waveguide TM mode fields

$$E_z^0(x,y) = E_0 \sin(\tfrac{m\pi}{a}x)\sin(\tfrac{n\pi}{b}y)$$
$$E_x^0(x,y) = -\tfrac{\gamma}{h^2}(\tfrac{m\pi}{a})E_0 \cos(\tfrac{m\pi}{a}x)\sin(\tfrac{n\pi}{b}y)$$
$$E_y^0(x,y) = -\tfrac{\gamma}{h^2}(\tfrac{n\pi}{b})E_0 \sin(\tfrac{m\pi}{a}x)\cos(\tfrac{n\pi}{b}y)$$
$$H_x^0(x,y) = \tfrac{j\omega\varepsilon}{h^2}(\tfrac{n\pi}{b})E_0 \sin(\tfrac{m\pi}{a}x)\cos(\tfrac{n\pi}{b}y)$$
$$H_y^0(x,y) = -\tfrac{j\omega\varepsilon}{h^2}(\tfrac{m\pi}{a})E_0 \cos(\tfrac{m\pi}{a}x)\sin(\tfrac{n\pi}{b}y)$$
$$\gamma = j\beta = j\sqrt{\omega^2\mu\varepsilon - (\tfrac{m\pi}{a})^2 - (\tfrac{n\pi}{b})^2}$$

Waveguide TE mode fields

$$H_z^0(x,y) = H_0 \cos(\tfrac{m\pi}{a}x)\cos(\tfrac{n\pi}{b}y)$$
$$E_x^0(x,y) = \tfrac{j\omega\mu}{h^2}(\tfrac{n\pi}{b})H_0 \cos(\tfrac{m\pi}{a}x)\sin(\tfrac{n\pi}{b}y)$$
$$E_y^0(x,y) = -\tfrac{j\omega\mu}{h^2}(\tfrac{n\pi}{b})H_0 \sin(\tfrac{m\pi}{a}x)\cos(\tfrac{n\pi}{b}y)$$
$$H_x^0(x,y) = \tfrac{\gamma}{h^2}(\tfrac{m\pi}{a})H_0 \sin(\tfrac{m\pi}{a}x)\cos(\tfrac{n\pi}{b}y)$$
$$H_y^0(x,y) = \tfrac{\gamma}{h^2}(\tfrac{n\pi}{b})H_0 \cos(\tfrac{m\pi}{a}x)\sin(\tfrac{n\pi}{b}y)$$
$$h^2 = (\tfrac{m\pi}{a})^2 + (\tfrac{n\pi}{b})^2$$