

1. (a) Consider the closed-loop system shown in Figure 1. Let  $\alpha$  be a parameter of  $G(s)$ . Also, let the sensitivities of  $G(s)$  and the closed-loop transfer function  $T(s)$  with respect to the parameter  $\alpha$  be

$$\text{defined by } S_{\alpha}^G \text{ and } S_{\alpha}^T, \text{ respectively. Show that } S_{\alpha}^T = \frac{1}{1+G(s)H(s)} \cdot S_{\alpha}^G. \quad (10\%)$$

- (b) Let  $G(s) = k/[s(s+\alpha)]$  and  $H(s) = 1$ , in Figure 1. We shall assume that the nominal value of the gain constant  $k$  is 10, and that of  $\alpha$  is 2. Determine the change in  $T(s)$  for a 5% change in the parameter  $\alpha$ . Calculate the value for  $\omega = 0.5$ , and hence determine the change in the steady-state response to

$$\text{the input } r(t) = 2\cos(0.5t). \text{ (Hint: } \frac{\Delta T(s)}{T(s)} \approx S_{\alpha}^T \cdot \frac{\Delta \alpha}{\alpha} \text{.)} \quad (15\%)$$

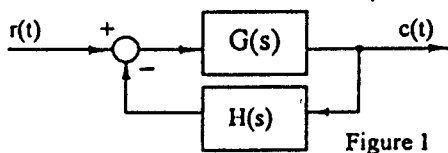


Figure 1

2. Consider the closed-loop system shown in Figure 1, where  $G(s) = 1/[s(1+0.5s)(1+s)]$  and  $H(s) = 1$ . Find the exact gain margin (GM), and determine the stability of the system based on the obtained gain margin. (15%)
3. A rotating load is connected to a field-controlled DC motor with negligible field inductance. A test results in the output load reaching a speed of 100 RPM within 0.1 second when a constant input of 100V is applied to the motor terminals. The output steady-state speed from the same test is found to be 300 RPM. Determine the transfer function  $\theta(s)/V_f(s)$  of the motor. (10%)
4. Consider the Type I system shown in Figure 2, where  $G(s) = 1/[s(s+1)]$ . We would like to design the compensation  $D(s)$  to meet the following requirements: (1) The steady-state value of  $y$  due to a constant unit disturbance  $w$  should be less than 0.8, and (2) the damping ratio  $\zeta = 0.7$ . Using root-locus techniques:
- (a) Show that proportional control alone is not adequate. (4%)
- (b) Show that proportional-derivative control will work. (4%)
- (c) Find values of the gains  $K$  and  $K_d$  for  $D(s) = K + K_d s$  that meet the design specifications. (12%)

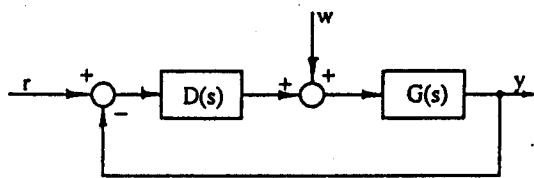


Figure 2

5. Consider the sampled-data system shown in Figure 3. Please determine  $C(z)$ . (15%)

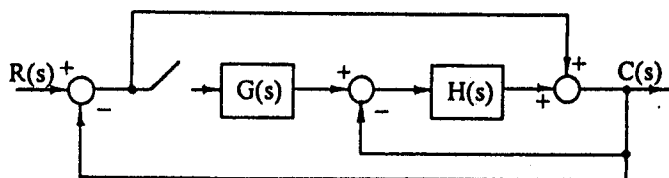


Figure 3

6. Consider a relaxed system whose input ( $u(t)$ ) and output ( $y(t)$ ) are related by  $y(t) = 0$ , for  $t \leq \alpha$  and  $y(t) = u(t)$ , for  $t > \alpha$ , for any  $u(t)$ , where  $\alpha$  is a fixed constant. Please explain and answer the following questions. Is the system linear? Is it time invariant? Is it causal? (15%)