國立成功大學 83學年度電研所碩考試(離散數學試題) 共 2 頁 第 1 頁

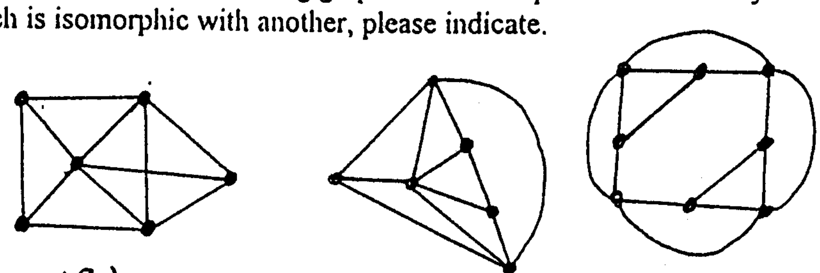
1: (10)
Suppose that a man hiked 6 miles the first hour and 4 miles the twelfth hour and hiked a total of 71 miles in 12 hours. Prove that he must have hiked at least 12 miles within a certain period of two consecutive hours.

2: (10)
Let A, B be sets with $|A|=m\geq n=|B|$, and let $a(m,n)$ count the number of onto functions from A to B. Show that

$$a(m,1)=1$$
$$a(m,n)=n^m-\sum_{i=1}^{n-1}\binom{n}{i}a(m,i),\ \text{when } m\geq n>1.$$

3: (15)
(a) If G1, G2 are (loop-free) undirected graphs, prove that G1, G2 are isomorphic if and only if $\overline{G1}$, $\overline{G2}$ are isomorphic.

(b) Determine whether the following graphs are isomorphic. If there is any graph which is isomorphic with another, please indicate.



(a) (b) (c)

4: (10)
Find $P(G,\lambda)$ for the graph G in Fig.3.

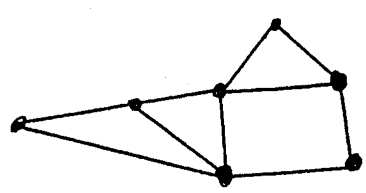


Fig.3

5: (10)
If T is a complete m-ary tree, the total path length of T is the sum of the lengths of all paths in the tree from the root to each of its vertices.
(a) For any nonnegative integer h, let x_h denote the minimal total path length for a complete m-ary tree of height h. Show that x_h satisfies the recurrence relation $x_{h+1}=x_h+m(h+1)$, with initial condition $x_0=0$.

(b) Solve the recurrence relation in part (a) for x_h.

6: (12)

For any $n \in Z$, $n \geq 0$, prove that

(a) $2^{2n+1}+1$ is divisible by 3.

(b) $\dfrac{n^7}{7} + \dfrac{n^3}{3} + \dfrac{11n}{21}$ is an integer.

7:(10)

(a) Find a recurrence relation for the number of ways to make a pile of n chips using green, gold, red, white, and blue chips such that no two gold chips are together. (b) Solve the recurrence relation.

8:(18)

Let $\Im = \{f: Z^+ \to R\}$. That is, $\Im$ is the set of all functions with domain Z^+ and codomain R.

(a) Define the relation $\Re$ on $\Im$ by $g \Re h$, for $g, h \in \Im$, if g is dominated by h and h is dominated by g. Prove that $\Re$ is an equivalence relation on $\Im$.

(b) For $f \in \Im$, let [f] denote the equivalence class of f for the relation $\Re$ of part(a). Let $\Im'$ be the set of equivalence classes induced by $\Re$. Define the relation ρ on $\Im'$ by $[g]\rho[h]$, for $[g], [h] \in \Im'$, if g is dominated by h. Verify that ρ is a partial order.

(c) For $\Re$ in part (a), let $f, f_1, f_2 \in \Im$ with $f_1, f_2 \in [f]$. If $f_1+f_2: Z^+ \to R$ is defined by $(f_1+f_2)(n) = f_1(n)+f_2(n)$, for $n \in Z^+$, prove or disprove that $f_1+f_2 \in [f]$.

9: (5)

Show that the following graph is self-dual.